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Instantons and matter in $\mathcal{N} = 1/2$ supersymmetric gauge theory

Simone Giombi, Riccardo Ricci, Daniel Robles-Llana and Diego Trancanelli

C.N. Yang Institute for Theoretical Physics
State University of New York at Stony Brook
Stony Brook, NY 11794-3840, U.S.A.
E-mail: sgiombi@insti.physics.sunysb.edu, rricci@insti.physics.sunysb.edu, daniel@insti.physics.sunysb.edu, dtrancan@insti.physics.sunysb.edu

Abstract: We extend the instanton calculus for $\mathcal{N} = 1/2$ U(2) supersymmetric gauge theory by including one massless flavor. We write the equations of motion at leading order in the coupling constant and we solve them exactly in the non(anti)commutativity parameter C. The profile of the matter superfield is deformed through linear and quadratic corrections in C. Higher order corrections are absent because of the fermionic nature of the back-reaction. The instanton effective action, in addition to the usual 't Hooft term, includes a contribution of order C^2 and is $\mathcal{N} = 1/2$ invariant. We argue that the $\mathcal{N} = 1$ result for the gluino condensate is not modified by the presence of the new term in the effective action.

Keywords: Nonperturbative Effects, Solitons Monopoles and Instantons, Superspaces.

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Contents

1. Introduction

Non-anticommutative superspace [1] has been thoroughly investigated recently [2], the interest being in part motivated by a surprising connection with string dynamics in non trivial backgrounds. Another reason of interest is the study of theories with reduced supersymmetry.

In non-anticommutative superspace half of the superspace fermionic variables are promoted to elements of a Clifford algebra. The complete set of anti-commutation relations is then

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} \qquad \{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = 0 \qquad \{\theta^\alpha, \bar\theta^{\dot\beta}\} = 0\,, \tag{1.1}$$

where $C^{\alpha\beta}$ is the deformation parameter. At the level of the supersymmetry algebra the only modification due to the *chiral* deformation $C^{\alpha\beta}$ is in the anti-commutator of the *anti-chiral* supercharges

$$\{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = -4C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta}\frac{\partial^2}{\partial y^\mu \partial y^\nu}\,, \tag{1.2}$$

where $y^\mu = x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$. The amount of supersymmetry left is therefore $\mathcal{N}=1/2$ and corresponds to the unbroken supersymmetry generators Q_α. The non trivial anti-commutator of the θ^α variables breaks also the Lorentz group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ to $\mathrm{SU}(2)_R$. One can write down a super Yang-Mills theory lagrangian which is invariant under $\mathcal{N} = 1/2$ supersymmetry only

$$\mathcal{L} = \frac{1}{g^2}\mathrm{Tr}\left(-\frac{1}{8}F_{\mu\nu}F^{\mu\nu} - \frac{i}{2}\bar\lambda\bar\sigma^\mu\mathcal{D}_\mu\lambda + \frac{1}{4}D^2 - iC^{\mu\nu}F_{\mu\nu}\bar\lambda\bar\lambda + \frac{|C|^2}{4}(\bar\lambda\bar\lambda)^2\right). \tag{1.3}$$

It is not hermitean and contains operators of dimensions 5 and 6 but still it is renormalizable [3].

As already remarked, a motivation for the study of theories in C-deformed superspace is given by their connection with strings propagating in geometries with non trivial background. More precisely, the superspace deformation (1.1) is related to $\mathcal{N}=2$ superstrings in euclidean R^4 with self-dual graviphoton field-strength background $F^{\dot\alpha\dot\beta} = 0$ [4, 2, 5]. The parameter $C^{\alpha\beta}$ and the selfdual field-strength are related as $(\alpha')^2 F^{\alpha\beta} = C^{\alpha\beta}$.[1]

Perturbative analysis of Wess-Zumino models and $\mathcal{N} = 1/2$ super Yang-Mills were performed in [2, 3] while instanton configurations were considered in [7, 8]. The presence of the deformation parameter $C^{\alpha\beta}$ alters the usual instanton profile. A nice feature of the C-deformed background is that it is fully solvable in the deformation: The equations of motion can be solved iteratively in the deformation parameter, the iteration in C terminating after a finite number of steps due to the fermionic nature of the back-reaction. The simplest non trivial case to study is when the gauge group is U(2) [7, 9, 10]. The *anti-self-dual* instanton configuration coincides with the familiar 't Hooft-Polyakov instanton while the *self-dual* equations of motion are modified by the presence of a fermionic source

$$F^+_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\bar\lambda\bar\lambda = 0\,. \tag{1.4}$$

The only novelty here is in the U(1) component of the gauge field which is linear in the deformation and quadratic in the Grassmann collective coordinates. The gluino zeromodes and the topological charge do not receive corrections in C. Generalization to U(N) gauge groups appeared in [10].

[1] For string theory related aspects see also [6].

A natural extension for obtaining more realistic theories is to include flavors by coupling $\mathcal{N} = 1/2$ U(2) super Yang-Mills to a matter superfield. The $\mathcal{N} = 1/2$ lagrangian in the presence of chiral superfields has been given in [11].

Adding flavors is useful because it allows one to work consistently in a semi-classical approximation by choosing the Higgs vacuum expectation value suitably larger than the strong dynamics scale of the gauge theory. This is the only regime where instanton methods are reliable.[2]

On the other hand, the incorporation of fundamental matter in instanton calculus substantially modifies the nature of the problem. In particular, it is known that additional zero-modes for the fundamental fermions appear in the Higgs phase and that the equations of motion can no longer be solved exactly but only order by order in g^2. The instanton configuration is therefore only an *approximate* solution. At leading order in the coupling constant the effects of matter are captured by additional terms in the instanton effective action which depend on the instanton size ρ, the asymptotic value of the Higgs field, and the extra fermionic collective coordinates in the matter sector. The presence of an effective potential which depends on the instanton moduli reflects the fact that we are not fully solving the equations of motion.

In this paper we study instanton solutions of $\mathcal{N} = 1/2$ U(2) super Yang-Mills with one massless flavor.[3] The matter sector is solved at leading order in perturbation theory by expanding around the pure $\mathcal{N}$=1/2 super Yang-Mills background. The new ingredients are the deformed U(1) connection and C-dependent Yukawa-like interaction terms. The strategy is then to reduce the equation of motions to non homogeneous Dirac and Laplace equations in the usual SU(2) undeformed instanton background. These equations are fully solved in C at the given leading order in g. We will see that the corrections to the matter fields will be only linear and quadratic in the deformation. Corrections of order C^3 do not appear due to the grassmannian nature of the iteration. We finally substitute the solutions in the action and evaluate their surface contribution to obtain the instanton effective action. The usual 't Hooft effective action gets a new contribution which is quadratic in C and quartic in the supersymmetry and superconformal collective coordinates. Still the presence of the new term does not modify the gluino condensate.

This paper is organized as follows: in sections 2 and 3 we review the $\mathcal{N} = 1/2$ superalgebra, the construction of the lagrangian, and the supersymmetry transformations of the fields. In section 4 we present the equations of motion. In section 5 we give the systematics of the expansion in the coupling constant while in section 6 we solve the equations of motion at each order in the deformation parameter C. In section 7 and 8 we obtain the instanton effective action and we discuss its supersymmetry properties. Finally, in section 9 we argue that the gluino condensate is not affected by the deformation. Notation and details of the calculations are collected in a series of appendices A–C.

[2]For a comprehensive review on these aspects see [12].

[3]Monopoles and vortices in a similar setting have been studied in [13].

JHEP10(2005)021

2. Review of $\mathcal{N} = 1/2$ supersymmetric theories

Four dimensional $\mathcal{N} = 1/2$ supersymmetric theories [2] arise when half of the fermionic coordinates of superspace θ^α obey a Clifford algebra

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} . \tag{2.1}$$

The non-trivial anticommutator (2.1) implies that functions of θ should be Weyl ordered. This is achieved by introducing a fermionic star product

$$f(\theta) \star g(\theta) = f(\theta) \exp\left(-\frac{C^{\alpha\beta}}{2} \frac{\overleftarrow{\partial}}{\partial\theta^\alpha} \frac{\overrightarrow{\partial}}{\partial\theta^\beta} \right) g(\theta) . \tag{2.2}$$

The rest of the coordinates obey their usual (anti)commutation relations

$$[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = [y^m, \bar{\theta}^{\dot\alpha}] = 0 , \tag{2.3}$$

where, in order to preserve the above relations and be able to properly define chiral and antichiral superfields, one has to work with the chiral coordinate $y^\mu = x^\mu + i\theta^\alpha \sigma^\mu_{\alpha\dot\alpha} \bar\theta^{\dot\alpha}$. This means that the supercharges take the form

$$\begin{aligned} Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} \\ \bar{Q}_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2i\theta^\alpha \partial_{\alpha\dot\alpha} \end{aligned} \tag{2.4}$$

and that the supersymmetry algebra is deformed to

$$\begin{aligned} \{Q_\alpha, Q_\alpha\} &= 0 \\ \{\bar{Q}_{\dot\alpha}, Q_\alpha\} &= 2i\partial_{\alpha\dot\alpha} \\ \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} &= -4C^{\alpha\beta} \partial_{\alpha\dot\alpha} \partial_{\beta\dot\beta} . \end{aligned} \tag{2.5}$$

Therefore, only half of the initial supersymmetries are realized linearly. Furthermore the rest of the (anti)commutators of the $\mathcal{N} = 1$ superconformal algebra which contain products of θ's are deformed, and the generators involved therein are no longer symmetries of the theory. The symmetry algebra that remains is the $\mathcal{N} = 1/2$ super Poincare algebra

$$\begin{aligned} [\bar{M}_{\dot\alpha\dot\beta}, \bar{M}_{\dot\gamma\dot\delta}] &= \epsilon_{\dot\alpha(\dot\gamma} \bar{M}_{\dot\delta)\dot\beta} + \epsilon_{\dot\beta(\dot\gamma} \bar{M}_{\dot\delta)\dot\alpha} \\ [\bar{M}_{\dot\alpha\dot\beta}, P_{\dot\gamma\dot\delta}] &= 4P_{\gamma(\dot\alpha}\epsilon_{\dot\beta)\dot\gamma} \\ [\bar{M}_{\dot\alpha\dot\beta}, \bar{S}_{\dot\gamma}] &= \epsilon_{(\dot\alpha\dot\gamma} \bar{S}_{\dot\beta)} \\ [P_{\alpha\dot\alpha}, \bar{S}_{\dot\beta}] &= 2i\epsilon_{\dot\alpha\dot\beta} Q_\alpha \\ [\tilde{D}, P_{\alpha\dot\alpha}] &= -iP_{\alpha\dot\alpha} \\ [\tilde{D}, \bar{S}_{\dot\alpha}] &= i\bar{S}_{\dot\alpha} , \end{aligned} \tag{2.6}$$

where the dilatation operator $\tilde{D}$ takes the form $\tilde{D} \equiv D - \frac{1}{2}R$ and accounts for the non trivial R-charge of C, which is given by -2.

One can write lagrangians in non(anti)commutative superspace. Note that the deformation (2.1) is chiral and therefore these theories are well defined only in euclidean space. The procedure is straightforward and consists in writing the usual $\mathcal{N} = 1$ supersymmetric action but replacing all products by star products. We do this in the next section.

JHEP10(2005)021

3. The lagrangian of $\mathcal{N} = 1/2$ super Yang-Mills with matter

In this paper we will be interested in $\mathcal{N} = 1/2$ U(2) super Yang-Mills coupled to fundamental matter, a theory which has been considered in [11]. The fundamental quark flavor corresponds to one chiral field in the **2** representation of U(2) and one chiral field in the $\bar{\mathbf{2}}$ representation of U(2). By writing the usual F- and D-terms for the usual undeformed $\mathcal{N} = 1$ theory and replacing ordinary products with star products (2.2) everywhere we obtain

$$\begin{aligned}\mathcal{L} = \int d^4\theta \;\; & \left[\Phi^\dagger \star e_\star^V \star \Phi \;+\; \tilde{\Phi} \star e_\star^{-V} \star \tilde{\Phi}^\dagger\right] + \\ & + i\tau \int d^2\theta \; \mathrm{Tr}\; W^\alpha \star W_\alpha - i\bar{\tau} \int d^2\bar{\theta} \; \mathrm{Tr}\; W^\dagger_{\dot{\alpha}} \star W^{\dagger\dot{\alpha}} \,, \end{aligned} \tag{3.1}$$

where the field-strengths are computed from the usual definition, but replacing ordinary products with star products as well.

As pointed out in previous literature, there are some difficulties concerning the definition of vector [2] and antichiral superfields [11]. These arise because of the fact that one would like infinitesimal gauge transformations

$$\begin{aligned} \delta\Phi &= -i\Lambda \star \Phi \,, & \delta\tilde{\Phi} &= i\tilde{\Phi} \star \Lambda \,, \\ \delta\Phi^\dagger &= i\Phi^\dagger \star \Lambda^\dagger \,, & \delta\tilde{\Phi}^\dagger &= -i\Lambda^\dagger \star \tilde{\Phi}^\dagger \,, \\ \delta e^V &= -i\Lambda^\dagger \star e_\star^V \;+\; ie_\star^V \star \Lambda \end{aligned} \tag{3.2}$$

to act as ordinary undeformed transformations on the component fields. This problem is surmounted by modifying the definition of the vector (in the Wess-Zumino gauge) and antichiral matter superfields from the undeformed case as

$$\begin{aligned} V(y,\theta,\bar{\theta}) &= -\theta\sigma^\mu\bar{\theta}A_\mu(y) + i\theta\theta\bar{\theta}\bar{\lambda}(y) - i\bar{\theta}\bar{\theta}\theta^\alpha\left(\lambda_\alpha(y) + \frac{1}{4}\epsilon_{\alpha\beta}C^{\beta\gamma}\sigma^\mu_{\gamma\dot{\gamma}}\{\bar{\lambda}^{\dot{\gamma}}, A_\mu\}(y)\right) + \\ &\quad + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left(D(y) - i\partial_\mu A^\mu(y)\right) \qquad (3.3) \\ \Phi^\dagger(\bar{y},\bar{\theta}) &= A^\dagger(\bar{y}) + \sqrt{2}\bar{\theta}\psi^\dagger(\bar{y}) + \bar{\theta}\bar{\theta}\left(F^\dagger(\bar{y}) + iC^{\mu\nu}\partial_\mu(A^\dagger A_\nu)(\bar{y}) - \frac{1}{4}C^{\mu\nu}A^\dagger A_\mu A_\nu(\bar{y})\right) \\ \tilde{\Phi}^\dagger(\bar{y},\bar{\theta}) &= \tilde{A}^\dagger(\bar{y}) + \sqrt{2}\bar{\theta}\tilde{\psi}^\dagger(\bar{y}) + \bar{\theta}\bar{\theta}\left(\tilde{F}^\dagger(\bar{y}) - iC^{\mu\nu}\partial_\mu(A_\nu\tilde{A}^\dagger)(\bar{y}) - \frac{1}{4}C^{\mu\nu}A_\mu A_\nu\tilde{A}^\dagger(\bar{y})\right), \end{aligned}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ is the ordinary chiral coordinate and $\bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$ is the antichiral coordinate.

In order for ordinary gauge transformations to preserve the C-deformed Wess-Zumino gauge for W_α, it can be shown that the usual chiral gauge parameter has to be modified as

$$\begin{aligned} \Lambda(y,\theta) &= -\phi(y) \\ \Lambda^\dagger(\bar{y},\bar{\theta}) &= -\phi(\bar{y}) - \frac{i}{2}\bar{\theta}\bar{\theta}C^{\mu\nu}\{\partial_\mu\phi, A_\nu\}(\bar{y})\,. \end{aligned} \tag{3.4}$$

One can then see that (3.2) and (3.4) reproduce ordinary gauge transformations for all component fields, namely

$$
\begin{aligned}
\delta A &= i\phi A\,, \qquad \delta\psi = i\phi\psi\,, \qquad \delta F = i\phi F\,,\\
\delta\tilde{A} &= -i\tilde{A}\phi\,, \qquad \delta\tilde{\psi} = -i\tilde{\psi}\phi\,, \qquad \delta\tilde{F} = -i\tilde{F}\phi\,,\\
\delta A_\mu &= -2\partial_\mu\phi + i[\phi, A_\mu]\,, \qquad \delta\lambda = i[\phi,\lambda]\,, \qquad \delta\bar{\lambda} = i[\phi,\bar{\lambda}]\,, \qquad \delta D = i[\phi, D]\,.
\end{aligned}
\tag{3.5}
$$

Finally, by expanding the star products, one can write (3.1) in components as

$$
\begin{aligned}
\mathcal{L} = {} & \mathrm{Tr}\left(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 2i\bar{\lambda}\bar{\sigma}^\mu\mathcal{D}_\mu\lambda + D^2\right) + \\
& + F^\dagger F - i\bar{\psi}\bar{\sigma}^\mu\mathcal{D}_\mu\psi - \mathcal{D}_\mu A^\dagger\mathcal{D}^\mu A + g^2A^\dagger DA + i\frac{\sqrt{2}}{2}g(A^\dagger\lambda\psi - \bar{\psi}\bar{\lambda}A) + \\
& + \tilde{F}\tilde{F}^\dagger - i\tilde{\psi}\sigma^\mu\mathcal{D}_\mu\bar{\tilde{\psi}} - \mathcal{D}_\mu\tilde{A}\mathcal{D}^\mu\tilde{A}^\dagger - g^2\tilde{A}D\tilde{A}^\dagger + i\frac{\sqrt{2}}{2}g\left(\tilde{A}\bar{\lambda}\bar{\tilde{\psi}} - \tilde{\psi}\lambda\tilde{A}^\dagger\right) + \\
& + \mathrm{Tr}\left(-iC^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda} + \frac{|C|^2}{4}(\bar{\lambda}\bar{\lambda})^2\right) + \\
& + iC^{\mu\nu}A^\dagger F_{\mu\nu}F - \frac{\sqrt{2}}{2}C^{\alpha\beta}\sigma^\mu_{\alpha\dot{\alpha}}\mathcal{D}_\mu A^\dagger\bar{\lambda}^{\dot{\alpha}}\psi_\beta - \frac{|C|^2}{4}A^\dagger\bar{\lambda}\bar{\lambda}F - \\
& - iC^{\mu\nu}\tilde{F}F_{\mu\nu}\tilde{A}^\dagger - \frac{\sqrt{2}}{2}C^{\alpha\beta}\sigma^\mu_{\alpha\dot{\alpha}}\tilde{\psi}_\beta\bar{\lambda}^{\dot{\alpha}}\mathcal{D}_\mu\tilde{A}^\dagger - \frac{|C|^2}{4}\tilde{F}\bar{\lambda}\bar{\lambda}\tilde{A}^\dagger\,.
\end{aligned}
\tag{3.6}
$$

In writing this expression we have rescaled the vector multiplet as $V\to gV$ and also the deformation parameter as $C^{\mu\nu}\to\frac{1}{g}C^{\mu\nu}$ so that the vector multiplet (3.3) is linear in g. The first three lines correspond to the undeformed F- and D-terms of the $\mathcal{N}=1$ lagrangian coupled to fundamental matter, whereas the last three lines give the C-deformations of the F- and D-terms. In our conventions $C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma^{\mu\nu\,\gamma}_{\alpha}$ and $|C|^2 = C_{\mu\nu}C^{\mu\nu} = 4\det C$. Moreover the covariant derivative is normalized as $\mathcal{D}_\mu = \partial_\mu + \frac{i}{2}A_\mu$.

The lagrangian (3.6) is invariant under a set of C-deformed supersymmetries generated by acting with Q_α on the various superfields of the theory. Because the deformed vector superfield contains an additional C-dependent term in the $\bar{\theta}^2\theta$ component the supersymmetry transformation of λ is deformed. Similarly the supersymmetry transformation of the $F^\dagger$ component of the antichiral superfields also contains an extra C-dependent term. By carrying out the usual procedure one finds the supersymmetries of the theory, which are given by

$$
\begin{aligned}
\delta A &= \sqrt{2}\epsilon\psi\,, \qquad \delta\psi_\alpha = \sqrt{2}\epsilon_\alpha F\,, \qquad \delta F = 0\\
\delta A^\dagger &= 0\,, \qquad \delta\bar{\psi}^{\dot{\alpha}} = -i\sqrt{2}\mathcal{D}_\mu A^\dagger(\epsilon\sigma^\mu)^{\dot{\alpha}}\\
\delta F^\dagger &= -i\sqrt{2}\mathcal{D}_\mu\bar{\psi}\bar{\sigma}^\mu\epsilon - iA^\dagger\epsilon\lambda + C^{\mu\nu}\left[\partial_\mu(A^\dagger\epsilon\sigma_\nu\bar{\lambda}) - \frac{i}{2}(A^\dagger\epsilon\sigma_\nu\bar{\lambda})A_\mu\right]\\
\delta A_\mu &= -i\bar{\lambda}\bar{\sigma}_\mu\epsilon\,, \qquad \delta D = -\epsilon\sigma^\mu\mathcal{D}_\mu\bar{\lambda}\\
\delta\lambda_\alpha &= i\epsilon_\alpha D + (\sigma^{\mu\nu}\epsilon)_\alpha\left(F_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\bar{\lambda}\bar{\lambda}\right), \qquad \delta\bar{\lambda}^{\dot{\alpha}} = 0\,,
\end{aligned}
\tag{3.7}
$$

JHEP10(2005)021

with similar expressions for the tilded fields. Note that we only wrote the chiral supersymmetries, as the anti-chiral ones are explicitely broken by the deformation. In addition, we see that only the tranformation of the antichiral auxiliary field and of the chiral gluino are modified from the usual case.

4. The equations of motion

The preliminary step before solving the equations of motion consists in eliminating the auxiliary fields from the action. This simplifies the lagrangian considerably. The $F^\dagger$ equation of motion implies $F=0$, which eliminates two of the C-deformed D-terms. Using the equations of motion for D, which are the same as in the undeformed case, we can write the lagrangian as

$$\begin{aligned}
\mathcal{L} = \operatorname{Tr}\left(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 2i\bar\lambda\bar\sigma^\mu\mathcal{D}_\mu\lambda\right) - \\
&- \mathcal{D}_\mu A^\dagger\mathcal{D}^\mu A - i\bar\psi\bar\sigma^\mu\mathcal{D}_\mu\psi + i\frac{\sqrt{2}}{2}g\left(A^\dagger\lambda\psi - \bar\psi\bar\lambda A\right) + \frac{1}{4}g^2\left(A^\dagger T^a A - \tilde{A}T^a\tilde{A}^\dagger\right)^2 - \\
&- \mathcal{D}_\mu\tilde{A}\mathcal{D}^\mu\tilde{A}^\dagger - i\tilde\psi\sigma^\mu\mathcal{D}_\mu\bar{\tilde\psi} + i\frac{\sqrt{2}}{2}g\left(\tilde{A}\bar\lambda\bar{\tilde\psi} - \tilde\psi\lambda\tilde{A}^\dagger\right) + \\
&+ \operatorname{Tr}\left(-iC^{\mu\nu}F_{\mu\nu}\bar\lambda\bar\lambda + \frac{|C|^2}{4}(\bar\lambda\bar\lambda)^2\right) - \\
&- \frac{\sqrt{2}}{2}C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu A^\dagger\bar\lambda^{\dot\alpha}\psi_\beta - \frac{\sqrt{2}}{2}C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\tilde\psi_\beta\bar\lambda^{\dot\alpha}\mathcal{D}_\mu\tilde{A}^\dagger\,.
\end{aligned} \tag{4.1}$$

We are now ready to write the equations of motion for the different fields. They read

$$\begin{aligned}
A^\nu &: \mathcal{D}_\mu\left(F^{\mu\nu} + iC^{\mu\nu}\bar\lambda\bar\lambda\right) + ig\{\bar\lambda_{\dot\alpha}\bar\sigma^{\nu\,\dot\alpha\beta},\lambda_\beta\} + g\bar\sigma^{\nu\,\dot\alpha\beta}\psi_\beta\bar\psi_{\dot\alpha} - 2igA^\dagger\mathcal{D}^\nu A + \\
&\qquad + \frac{\sqrt{2}}{2}iC^{\alpha\beta}\sigma^\nu_{\alpha\dot\alpha}(\bar\lambda^{\dot\alpha}\psi_\beta A^\dagger + \tilde\psi_\beta\bar\lambda^{\dot\alpha}\tilde{A}^\dagger) - g\bar\sigma^{\nu\,\dot\alpha\beta}\bar{\tilde\psi}_{\dot\alpha}\tilde\psi_\beta - 2ig\tilde{A}\mathcal{D}^\nu\tilde{A}^\dagger = 0\,,\\
\lambda &: \bar\sigma^{\mu\,\dot\alpha\alpha}\mathcal{D}_\mu\lambda_\alpha + \bar\lambda^{\dot\alpha}\left(C_{\mu\nu}F^{\mu\nu} + i\frac{|C|^2}{2}\bar\lambda\bar\lambda\right) + \frac{1}{\sqrt{2}}gA\bar\psi^{\dot\alpha} - \frac{1}{\sqrt{2}}g\bar{\tilde\psi}^{\dot\alpha}\tilde{A} - \\
&\qquad - i\frac{1}{\sqrt{2}}C^{\alpha\beta}\sigma^{\mu\ \dot\alpha}_{\alpha}[\psi_\beta\mathcal{D}_\mu A^\dagger + (\mathcal{D}_\mu\tilde{A}^\dagger)\tilde\psi_\beta] = 0\,,\\
\bar\lambda &: \sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu\bar\lambda^{\dot\alpha} + \frac{g}{\sqrt{2}}\psi_\alpha A^\dagger - \frac{g}{\sqrt{2}}\tilde{A}^\dagger\tilde\psi_\alpha = 0\,,\\
A &: \mathcal{D}^2 A + i\frac{\sqrt{2}}{2}g\lambda\psi + \frac{1}{2}g^2(A^\dagger T^a A - \tilde{A}T^a\tilde{A}^\dagger)T^a A + \sqrt{2}C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu(\bar\lambda^{\dot\alpha}\psi_\beta) = 0\,,\\
A^\dagger &: \mathcal{D}^2 A^\dagger - i\frac{\sqrt{2}}{2}g\bar\psi\bar\lambda + \frac{1}{2}g^2(A^\dagger T^a A - \tilde{A}T^a\tilde{A}^\dagger)A^\dagger T^a = 0\,,\\
\psi &: \bar\sigma^{\mu\,\dot\alpha\alpha}\mathcal{D}_\mu\psi_\alpha + \frac{\sqrt{2}}{2}g\bar\lambda^{\dot\alpha}A = 0\,,\\
\bar\psi &: \sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu\bar\psi^{\dot\alpha} - \frac{\sqrt{2}}{2}gA^\dagger\lambda_\alpha - i\frac{\sqrt{2}}{2}C_\alpha^{\beta}\sigma^\mu_{\beta\dot\beta}(\mathcal{D}_\mu A^\dagger)\bar\lambda^{\dot\beta} = 0\,,
\end{aligned} \tag{4.2}$$

and similar equations for $\tilde{A}$, $\tilde{A}^\dagger$, $\tilde\psi$, and $\bar{\tilde\psi}$.

5. Expansion in g of the equations of motion

In usual $\mathcal{N}=1$ instanton calculations with matter fields there are no exact solutions to the equations of motion when the lowest components of the fundamental chiral fields acquire vacuum expectation values. The next best thing to do, if one does not want to use constrained instantons [14], is to solve the equations approximately to leading order in the coupling constant. To that purpose one solves for the matter fields around the approximate solutions[4]

$$F^{+}_{\mu\nu}=0 \qquad {}^{(0)}\mathcal{D}^2 A = 0\,, \tag{5.1}$$

where the covariant Laplace equation is taken in the instanton background. By appropriate partial integration of the kinetic term for the chiral scalars one can capture the leading effect due to the non-vanishing Yukawa terms.

We will demonstrate that the same procedure holds if one expands around the deformed instanton solutions of $\mathcal{N}=1/2$ super Yang-Mills. The leading effect of this deformation, for the U(2) case, is to modify the zero-mode equation for the fundamental fermions and the covariant Laplace equations for the scalars.

5.1 Systematics of the expansion in the coupling constant

As a first step we proceed to set up a systematic expansion for solving the equations of motion order by order in the coupling constant. To lowest order the equations in the pure gauge sector are

$$\begin{aligned}
A^{\nu} &: \mathcal{D}_{\mu}\left(F^{\mu\nu}+iC^{\mu\nu}\bar\lambda\bar\lambda\right)=0\\
\lambda &: \bar\sigma^{\mu\,\dot\alpha\alpha}\mathcal{D}_{\mu}\lambda_{\alpha}+\bar\lambda^{\dot\alpha}\left(C_{\mu\nu}F^{\mu\nu}+i\frac{|C|^2}{2}\bar\lambda\bar\lambda\right)=0\\
\bar\lambda &: \sigma^{\mu}_{\alpha\dot\alpha}\mathcal{D}_{\mu}\bar\lambda^{\dot\alpha}=0
\end{aligned} \tag{5.2}$$

whose solution was studied in [7, 9, 10]. For the U(2) theory that we are considering in this paper the solutions can be written in the form

$$\begin{aligned}
A_{\alpha\dot\alpha} &= g^{-1}\left(A^{\mathrm{SU}(2)}_{\alpha\dot\alpha}+A^{C}_{\alpha\dot\alpha}\right)\\
\bar\lambda^{\dot\alpha} &= g^{-1/2}\bar\lambda^{\mathrm{SU}(2)\,\dot\alpha}\,, \qquad \lambda_{\alpha}=0\,,
\end{aligned} \tag{5.3}$$

where $A^{\mathrm{SU}(2)}$ and $\bar\lambda^{\mathrm{SU}(2)}$ are the usual solutions for the undeformed SU(2) theory

$$\begin{aligned}
A^{\mathrm{SU}(2)\,\dot a\dot b}_{\alpha\dot\alpha}(x,x_0,\rho) &= \frac{-2i}{(x-x_0)^2+\rho^2}\left(\delta^{\dot a}_{\dot\alpha}(x-x_0)^{\dot b}_{\alpha}+\delta^{\dot b}_{\dot\alpha}(x-x_0)^{\dot a}_{\alpha}\right)\\
\bar\lambda^{\mathrm{SU}(2)\,\dot a\dot b}_{\dot\alpha}(x,x_0,\rho,\bar\xi) &= \frac{8i\rho^2}{[(x-x_0)^2+\rho^2]^2}\left(\delta^{\dot a}_{\dot\alpha}\delta^{\dot b}_{\dot\beta}+\delta^{\dot b}_{\dot\alpha}\delta^{\dot a}_{\dot\beta}\right)\bar\xi^{\dot\beta}\,,
\end{aligned} \tag{5.4}$$

where $\bar\xi^{\dot\beta}=\bar\zeta^{\dot\beta}+x^{\dot\beta}{}_{\alpha}\eta^{\alpha}$ is the supersymmetric fermionic collective coordinate. A^{C} is a C-dependent correction which affects only the U(1) subgroup of U(2). It is given by [9, 10]

$$A^{C}_{\alpha\dot\alpha}(x,x_0,\rho,\bar\zeta,\eta)=-4iC_{\alpha}{}^{\beta}\partial_{\beta\dot\alpha}(2\bar\zeta^2K_1+2\rho^2\eta^2K_2+4\rho^2K_3)\,, \tag{5.5}$$

[4] A superscript index on the left indicates the order of the expansion in the coupling constant. Later we will use superscripts on the right to indicate the order of the expansion in the C-deformation.

with

$$
\begin{aligned}
K_1(x,x_0,\rho) &= \frac{(x-x_0)^2}{[(x-x_0)^2+\rho^2]^2} - \frac{2}{(x-x_0)^2+\rho^2} \\
K_2(x,x_0,\rho) &= \frac{(x-x_0)^2}{[(x-x_0)^2+\rho^2]^2} + \frac{1}{(x-x_0)^2+\rho^2} \\
K_3(x,x_0,\rho) &= \frac{\bar\zeta_{\dot\alpha}\eta_\alpha (x-x_0)^{\dot\alpha\alpha}}{[(x-x_0)^2+\rho^2]^2}\,.
\end{aligned} \tag{5.6}
$$

Explicitly this deformed connection reads

$$
A^C_{\alpha\dot\alpha} = A^{(\bar\zeta^2)}_{\alpha\dot\alpha} + A^{(\eta^2)}_{\alpha\dot\alpha} + A^{(\bar\zeta\eta)}_{\alpha\dot\alpha}\,, \tag{5.7}
$$

where [5]

$$
\begin{aligned}
A^{(\bar\zeta^2)}_{\alpha\dot\alpha} &= 16iC_\alpha{}^\beta x_{\beta\dot\alpha}\frac{x^2+3\rho^2}{(x^2+\rho^2)^3}\bar\zeta^2 \\
A^{(\eta^2)}_{\alpha\dot\alpha} &= -32i\rho^2\, C_\alpha{}^\beta x_{\beta\dot\alpha}\frac{x^2}{(x^2+\rho^2)^3}\eta^2 \\
A^{(\bar\zeta\eta)}_{\alpha\dot\alpha} &= -32i\rho^2 C_\alpha{}^\beta\bar\zeta_{\dot\gamma}\eta_\gamma\frac{2x^{\dot\gamma\gamma}x_{\beta\dot\alpha} - \delta^\gamma_\beta\delta^{\dot\gamma}_{\dot\alpha}(x^2+\rho^2)}{(x^2+\rho^2)^3}\,.
\end{aligned} \tag{5.8}
$$

From inspection of the action and of the equations of motion one arrives at the following standard g-expansion of the various fields [12]

$$
\begin{aligned}
A_\mu &= g^{-1}\,{}^{(0)}A_\mu + g\,{}^{(1)}A_\mu + \cdots \\
\bar\lambda &= g^{-1/2}\,{}^{(0)}\bar\lambda + g^{3/2}\,{}^{(1)}\bar\lambda + \cdots \qquad \lambda = g^{1/2}\,{}^{(0)}\lambda + g^{5/2}\,{}^{(1)}\lambda + \cdots \\
\bar\psi &= g^{-1/2}\,{}^{(0)}\bar\psi + g^{3/2}\,{}^{(1)}\bar\psi + \cdots \qquad \psi = g^{1/2}\,{}^{(0)}\psi + g^{5/2}\,{}^{(1)}\psi + \cdots \\
A^\dagger &= g^{0}\,{}^{(0)}A^\dagger + g^2\,{}^{(1)}A^\dagger + \cdots \qquad A = g^{0}\,{}^{(0)}A + g^2\,{}^{(1)}A + \cdots\,.
\end{aligned} \tag{5.9}
$$

We remark here that, as in the undeformed case, there are no non-trivial solutions for the equation for the ψ zero-modes [6]

$$
\bar\sigma^{\mu\,\dot\alpha\alpha}\mathcal{D}_\mu\psi_\alpha = 0\,. \tag{5.10}
$$

Therefore the g expansion for ψ starts at order 1/2. Usually one drops this field because its contribution to the action through the term $gA^\dagger\lambda\psi$ is not leading order. However we need to include ψ in the present case because it contributes to the leading order effective action through the C-dependent Yukawa-like interaction.

5.2 $\mathcal{N}=1/2$ with one flavor

In the absence of an instanton background, the $\mathcal{N}=1/2$ theory has flat directions in field space. These directions are found by solving the D-flatness equations

$$
D^a = \left(A^\dagger T^a A - \tilde A T^a\tilde A^\dagger\right) = 0. \tag{5.11}
$$

[5] For simplicity, we set from this moment $x_0=0$.

[6] Writing $\bar{\mathcal{D}} = \bar{\mathcal{D}}^{(0)} + A^C$ and $\psi = \psi^{(0)} + \psi^{(C)}$ the equation at order C^0 gives just $\bar{\mathcal{D}}^{(0)}\psi^{(0)}=0$, which implies $\psi^{(0)}=0$ because $\bar{\mathcal{D}}^{(0)}$ is just the usual Dirac operator in the instanton background, which has no zero-modes. Then at order C one has $\bar{\mathcal{D}}^{(0)}\psi^C=0$ which in turn gives $\psi^C=0$.

JHEP10(2005)021

We denote the vacuum expectation values of A and $\tilde{A}$ with q and $\tilde{q}$. They are solutions to (5.11) and read

$$q^{\alpha} = \begin{pmatrix} q \\ 0 \end{pmatrix} \qquad \tilde{q}_{\alpha} = (\, \tilde{q} \quad 0\,) \tag{5.12}$$

with $|q|^2 = |\tilde{q}|^2$.

Now, as soon as the instanton background is switched on the Dirac operator for the fundamental fermions has zero-modes, which are deformed through C-dependent terms. These feed on the right-hand side of the covariant Laplace equation for $A^\dagger$. One then ends up having to solve the coupled equations

$$\begin{aligned} \mathcal{D}^2 A &= -\frac{\sqrt{2}}{2} C^{\alpha\beta} \sigma^{\mu}_{\alpha\dot{\alpha}} \mathcal{D}_{\mu} (\bar{\lambda}^{\dot{\alpha}} \psi_{\beta}) \,, \\ \mathcal{D}^2 A^\dagger &= i \frac{\sqrt{2}}{2} g \bar{\psi} \bar{\lambda} \,, \\ \bar{\mathcal{D}}^{\dot{\alpha}\alpha} \psi_{\alpha} &= -\frac{\sqrt{2}}{2} g \bar{\lambda}^{\dot{\alpha}} A \,, \\ \mathcal{D}_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} &= i \frac{\sqrt{2}}{2} C_{\alpha}^{\ \beta} \sigma^{\mu}_{\beta\dot{\beta}} (\mathcal{D}_{\mu} A^\dagger) \bar{\lambda}^{\dot{\beta}} \,, \end{aligned} \tag{5.13}$$

where the covariant derivative is with respect to the deformed connection given in (5.3). The equations for the scalar fields must be supplemented with the boundary conditions at infinity $A \to q$, $\tilde{A} \to \tilde{q}$. There are similar equations also for the tilded fields.

We stress now that in our solutions we set the matter auxiliary fields $F = F^\dagger = 0$, and these conditions are preserved under a chiral supersymmetry transformation. For F this is obvious, as the transformation is just $\delta_\epsilon F = 0$. For $F^\dagger$ the order g^0 part of the $\mathcal{N} = 1/2$ transformation is proportional to the deformed equation of motion for $\bar{\psi}$, and thus is also zero for our solution. Similarly, the $D = 0$ constraint is preserved on-shell.

6. Solution to the leading order coupled equations

We saw in section 5 that in order to consistently couple the pure $\mathcal{N} = 1/2$ super Yang-Mills instanton to matter to leading order in the coupling constant one had to solve (5.13). We rewrite them in two-component spinor notation, abandoning sigma matrices[7]

$$\begin{aligned} \mathcal{D}^2 A &= -h \frac{\sqrt{2}}{2} C^{\alpha\beta} \mathcal{D}_{\alpha\dot{\alpha}} (\bar{\lambda}^{\dot{\alpha}} \psi_{\beta}) \\ \mathcal{D}^2 A^\dagger &= i \frac{\sqrt{2}}{2} g \bar{\psi}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} \\ \bar{\mathcal{D}}^{\dot{\alpha}\alpha} \psi_{\alpha} &= -\frac{\sqrt{2}}{2} g \bar{\lambda}^{\dot{\alpha}} A \\ \mathcal{D}_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} &= ih \frac{\sqrt{2}}{2} C_{\alpha}^{\beta} (\mathcal{D}_{\beta\dot{\beta}} A^\dagger) \bar{\lambda}^{\dot{\beta}} \,, \end{aligned} \tag{6.1}$$

[7] We refer to the appendix A for our conventions.

JHEP10(2005)021

where $\mathcal{D}$ is taken with respect to the deformed instanton connection and reads explicitly

$$\mathcal{D}_{\alpha\dot{\alpha}} = \partial_{\alpha\dot{\alpha}} + \frac{i}{2} A^{\mathrm{SU}(2)}_{\alpha\dot{\alpha}} + \frac{ik}{2} A^{C}_{\alpha\dot{\alpha}} \equiv \mathcal{D}^{(0)}_{\alpha\dot{\alpha}} + \frac{ik}{2} A^{C}_{\alpha\dot{\alpha}} \,. \tag{6.2}$$

In (6.1) and (6.2) we have introduced two book-keeping parameters h and k. This helps in keeping track of the contributions coming from the U(1) part of the connection and the new Yukawa-like C-dependent interaction. In the final results reported in the main text we set $k = 1$ and $h = 1$, as required by supersymmetry of the action (4.1). On the other hand, we keep k and h explicit in the appendix B to stress the remarkable simplifications occurring when $k = 1 = h$.

We will solve (6.1) order by order in C. To do so we write the different fields as an expansion in C

$$\begin{aligned} A &= A^{(0)} + A^{(1)} + \cdots , \qquad & \tilde{A} &= \tilde{A}^{(0)} + \tilde{A}^{(1)} + \cdots \\ A^{\dagger} &= A^{\dagger(0)} + A^{\dagger(1)} + \cdots , \qquad & \tilde{A}^{\dagger} &= \tilde{A}^{\dagger(0)} + \tilde{A}^{\dagger(1)} + \cdots , \\ \bar{\psi} &= \bar{\psi}^{(0)} + \bar{\psi}^{(1)} + \ldots , \qquad & \bar{\tilde{\psi}} &= \bar{\tilde{\psi}}^{(0)} + \bar{\tilde{\psi}}^{(1)} + \cdots , \end{aligned} \tag{6.3}$$

where the superscript counts the power of C.

The general strategy to solve (6.1) is to recast them in the form of Poisson equations in the background of the undeformed SU(2) instanton. Generically we will encounter equations of the form

$$(\mathcal{D}^{(0)})^2 \; \Phi = \mathcal{J} \,, \tag{6.4}$$

where $\mathcal{J}$ may contain contributions from the deformed connection A^C. The solution to (6.4) is given by the sum of the solution to the homogeneous equation and a particular solution. The latter is obtained by inverting the laplacian with the appropriate propagator [15] , given in the appendix A.

6.1 Zero-th order in C

At zero-th order in C the equations to solve are

$$\begin{aligned} (\mathcal{D}^{(0)})^2 A^{(0)} &= 0 \\ (\mathcal{D}^{(0)})^2 A^{\dagger(0)} &= i\frac{\sqrt{2}}{2} g \bar{\psi}^{(0)}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} \\ \bar{\mathcal{D}}^{(0)\dot{\alpha}\alpha} \psi_{\alpha(0)} &= -\frac{\sqrt{2}}{2} g \bar{\lambda}^{\dot{\alpha}} A^{(0)} \\ \mathcal{D}^{(0)}_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}(0)} &= 0 \,, \end{aligned} \tag{6.5}$$

and similarly for the tilded fields. These equations are the usual ones that one encounters in $\mathcal{N} = 1$ supersymmetric QCD and have already been studied in the literature (see for example [16, 12]).

The details of our computations are contained in the appendix B. Here we only present the final results.

JHEP10(2005)021

The solution to the first equation in (6.5) has to be subject to the boundary condition $A \to q$ at infinity in order to satisfy (5.11). One finds that

$$A^{\dot a(0)}(x,\rho) = q^{\alpha}\frac{x^{\dot a}_{\alpha}}{\sqrt{x^2+\rho^2}}\,. \tag{6.6}$$

We remind that q^{α} is a constant two-components vector which is set to $q^{\alpha} = q\delta^{\alpha}_{1}$ by the D-term constraint (5.11).

The next equation one can solve is the equation for the fundamental fermionic zero-mode $\bar\psi^{(0)}$. Its solution is

$$\bar\psi^{(0)}_{\dot\alpha\dot a}(x,\rho,\mathcal{K}) = g^{-1/2}\frac{\rho^2\epsilon_{\dot a\dot\alpha}}{(x^2+\rho^2)^{3/2}}\mathcal{K}\,, \tag{6.7}$$

where $\mathcal{K}$ is a fermionic fundamental collective coordinate.

Having obtained (6.7) one can solve the second equation in (6.5) by inverting the covariant laplacian and adding the solution of the homogeneous equation, which is essentially (6.6). This procedure yields

$$A^{\dagger(0)}_{\dot a}(x,\rho,\mathcal{K},\bar\xi) = q^{\dagger}_{\alpha}\frac{x^{\alpha}_{\dot a}}{\sqrt{x^2+\rho^2}} - \frac{\sqrt{2}\rho^2\mathcal{K}\bar\xi_{\dot a}}{(x^2+\rho^2)^{3/2}}\,, \tag{6.8}$$

where $\bar\xi^{\dot a} = \bar\zeta^{\dot a} + x^{\dot a}_{\alpha}\eta^{\alpha}$ are the adjoint fermionic collective coordinates inherited from $\bar\lambda$.

Finally, the solution for the fundamental fermionic field $\psi^{(0)}$ is[8]

$$\psi^{\alpha\dot a(0)}(x,\rho,\bar\xi) = -g^{1/2}\frac{2\sqrt{2}i\rho^2}{(x^2+\rho^2)^{3/2}}q^{\alpha}\bar\xi^{\dot a}\,. \tag{6.9}$$

This equation is solved by acting on both sides of it with a covariant derivative $\mathcal{D}^{(0)}_{\beta\dot\alpha}$ and then solving the resulting Poisson equation [9]. In this case there is no solution to the homogeneous equation because of the self-duality of the instanton background we have chosen.

Before moving to the first order, we briefly comment on how the structure of the solutions is dictated by supersymmetry. This offers an alternative way to derive these solutions. In a purely bosonic background, the bosonic zero modes are

$$A^{(0)\dot a} = \frac{q^{\alpha}x^{\dot a}_{\alpha}}{\sqrt{x^2+\rho^2}} \qquad A^{\dagger(0)}_{\dot a} = \frac{q^{\dagger}_{\alpha}x^{\alpha}_{\dot a}}{\sqrt{x^2+\rho^2}}\,. \tag{6.10}$$

The solution for $\psi^{(0)}$ is obtained by applying the broken supercharge $\bar Q_{\dot\alpha}$ with the collective coordinate $\bar\zeta^{\dot\alpha}$ as supersymmetry parameter

$$\psi^{\alpha\dot a} = \delta_{\bar Q}\psi^{\alpha\dot a} \sim \bar\zeta_{\dot\alpha}\left(\bar{\mathcal{D}}^{\dot\alpha\alpha}A\right)^{\dot a} \sim \frac{q^{\alpha}\bar\zeta^{\dot a}}{(x^2+\rho^2)^{3/2}}\,. \tag{6.11}$$

[8]Note that in the usual undeformed case $\psi^{(0)}$ is taken to be zero, since $\bar{\mathcal{D}}^{(0)}$ has no zero-modes.

[9]We use $\mathcal{D}^{(0)}_{\beta\dot\alpha}\bar{\mathcal{D}}^{(0)\dot\alpha\alpha} = -\delta^{\alpha}_{\beta}(\mathcal{D}^{(0)})^2$. Note that there is no field-strenght piece because self-duality of the instanton background implies $F^{(0)\alpha}{}_{\beta} = 0$.

JHEP10(2005)021

We saw in the previous paragraph that the origin of this solution is the Yukawa interaction in the ψ equation since $\bar{\mathcal{D}}^{(0)\dot\alpha\alpha}$ has no normalizable zero-modes. In the SU(2) self-dual background there are two other fundamental fermionic zero-modes which are generated by applying the broken superconformal generator S^α with η as superconformal parameter. This amounts to replacing $\bar\zeta \to \bar\xi$ and thus gives (6.9). The antichiral fermionic solution can be found observing that, in presence of a non-vanishing vacuum expectation value q also the *chiral* supersymmetry generators act not trivially. The corresponding transformation parameter gets identified with the fundamental fermionic collective coordinate $\mathcal{K}_\alpha$ and the fermionic solution reads

$$\bar\psi_{\dot\alpha\dot a} \equiv \delta_Q \bar\psi_{\dot\alpha\dot a} \sim \mathcal{K}^\alpha \left(\mathcal{D}_{\alpha\dot\alpha} A^\dagger\right)_{\dot a} \sim \frac{\epsilon_{\dot a\dot\alpha}\rho^2 q^\dagger_\alpha \mathcal{K}^\alpha}{(x^2+\rho^2)^{3/2}} \,. \tag{6.12}$$

This is the zero mode of the $\mathcal{D}^{(0)}_{\alpha\dot\alpha}$ operator given in (6.7), once we identify $\mathcal{K} \equiv q^\dagger_\alpha \mathcal{K}^\alpha$. With the antichiral fermion switched on we generate a new contribution to $A^\dagger$ using the broken supercharge $\bar{Q}_{\dot\alpha}$

$$A^{\dagger new}_{\dot a} = A^\dagger_{\dot a} + \delta_{\bar Q} A^\dagger_{\dot a} = q^\dagger_\alpha \frac{x^\alpha_{\dot a}}{\sqrt{x^2+\rho^2}} - \bar\zeta_{\dot a} \frac{\sqrt{2}\rho^2 \mathcal{K}}{(x^2+\rho^2)^{3/2}} \,. \tag{6.13}$$

As before $\bar\zeta \to \bar\xi$ applying also the broken superconformal generator S.

6.2 First order in C

We now move on to the first order in the deformation parameter. The corresponding equations are

$$\begin{aligned}
&\bar{\mathcal{D}}^{\dot\alpha\alpha(0)} \mathcal{D}^{(0)}_{\alpha\dot\alpha} A^{(1)} + igk A^{\dot\alpha\alpha(1)} \mathcal{D}^{(0)}_{\alpha\dot\alpha} A^{(0)} = h\sqrt{2} C^{\alpha\beta} \mathcal{D}^{(0)}_{\alpha\dot\alpha} (\bar\lambda^{\dot\alpha} \psi^{(0)}_\beta) \\
&\bar{\mathcal{D}}^{\dot\alpha\alpha(0)} \mathcal{D}^{(0)}_{\alpha\dot\alpha} A^{\dagger(1)} - igk A^{\dot\alpha\alpha(1)} \mathcal{D}^{(0)}_{\alpha\dot\alpha} A^{\dagger(0)} = -i\sqrt{2} g \bar\psi^{(1)}_{\dot\alpha} \bar\lambda^{\dot\alpha} \\
&\bar{\mathcal{D}}^{\dot\alpha\alpha(0)} \psi^{(1)}_\alpha + \frac{ig}{2} k A^{\dot\alpha\alpha(1)} \psi^{(0)}_\alpha = -\frac{\sqrt{2}}{2} g \bar\lambda^{\dot\alpha} A^{(1)} \\
&\mathcal{D}^{(0)}_{\alpha\dot\alpha} \bar\psi^{\dot\alpha(1)} - \frac{ig}{2} k A^{(1)}_{\alpha\dot\alpha} \bar\psi^{\dot\alpha(0)} = ih\frac{\sqrt{2}}{2} C_\alpha^{\ \beta} (\mathcal{D}^{(0)}_{\beta\dot\beta} A^{\dagger(0)}) \bar\lambda^{\dot\beta} \,.
\end{aligned} \tag{6.14}$$

Again, the reader is referred to the appendix B for details of the computations.

The first order correction to the fundamental scalar $A^{(1)}$ is quadratic in the grassmannian collective coordinates and turns out to be

$$A^{(1)\dot a} = \frac{2\rho^2 q_\alpha C^{\alpha\beta}}{(x^2+\rho^2)^{5/2}} \left[x^{\dot a}_\beta \left(\bar\zeta^2 - 2\bar\zeta_{\dot\gamma} x^{\dot\gamma\gamma} \eta_\gamma + \rho^2 \eta^2 \right) + 2(x^2+\rho^2) \bar\zeta^{\dot a} \eta_\beta \right] . \tag{6.15}$$

The equation for the spinor $\bar\psi^{(1)}$ can be converted into a Poisson equation by introducing a fermionic prepotential via the Ansatz

$$\bar\psi^{\dot\alpha(1)} = \bar{\mathcal{D}}^{\dot\alpha\beta} \Psi^{(1)}_\beta \tag{6.16}$$

and solving for $\Psi^{(1)}$.[10]

[10] In this case it is not convenient to act on the equation with a covariant derivative because this would produce a curvature piece since $\bar{\mathcal{D}}^{(0)\dot\beta\alpha} \mathcal{D}^{(0)}_{\alpha\dot\alpha} \sim -\delta^{\dot\alpha}_{\dot\beta} (\mathcal{D}^{(0)})^2 + F^{(0)\dot\alpha}{}_{\dot\beta}$.

JHEP10(2005)021

The final solution for $\bar\psi^{(1)}$ consists of two contributions proportional to $\mathcal{K}$ and q

$$\bar\psi^{(1)} = \bar\psi^{(1)}_{\mathcal{K}} + \bar\psi^{(1)}_{q}\,, \tag{6.17}$$

where

$$\begin{aligned}
(\bar\psi^{(1)}_{\mathcal{K}})^{\dot\alpha}_{\dot a} &= g^{-1/2}\frac{4\mathcal{K}\rho^2 C_\alpha{}^{\beta} x^{\dot\alpha\alpha}}{(x^2+\rho^2)^{7/2}}\Big[x_{\beta\dot a}\left(\bar\zeta^2 - 2\bar\zeta_{\dot\gamma}x^{\dot\gamma\gamma}\eta_\gamma + \rho^2\eta^2\right) + 2(x^2+\rho^2)\bar\zeta_{\dot a}\eta_\beta\Big]\\
(\bar\psi^{(1)}_{q})^{\dot\alpha}_{\dot a} &= g^{-1/2}\frac{4\sqrt{2}q^\dagger_\alpha C^{\alpha\beta}\rho^2}{(x^2+\rho^2)^{5/2}}\Big[x^{\dot\alpha}_{\beta}\bar\xi_{\dot a} - \delta^{\dot\alpha}_{\dot a}(x_{\beta\dot\beta}\bar\zeta^{\dot\beta} + \rho^2\eta_\beta)\Big]\,.
\end{aligned} \tag{6.18}$$

With the latter result one can compute all the currents in the second equation of (6.14) and finally obtain the first order correction in C to the antifundamental scalar $A^{\dagger(1)}$

$$A^{\dagger(1)}_{\dot a} = \frac{2\rho^2 q^\dagger_\alpha C^{\alpha\beta}}{(x^2+\rho^2)^{5/2}}\Big[x_{\beta\dot a}\left(\bar\zeta^2 - 2\bar\zeta_{\dot\gamma}x^{\dot\gamma\gamma}\eta_\gamma + \rho^2\eta^2\right) + 2(x^2+\rho^2)\bar\zeta_{\dot a}\eta_\beta\Big]\,. \tag{6.19}$$

Some remarks are now in order. From the solution (B.19) in the appendix, one notices that cancellations at $h = 1 = k$ imply that $A^{\dagger(1)}$ behaves as $1/x^3$ at infinity. Therefore it will not contribute to the effective action as we will explain in section 7. Moreover, the part proportional to $\mathcal{K}$ vanishes at $h = 1 = k$. Finally, we observe that the solutions for $A^{(1)}$ and $A^{\dagger(1)}$ are conjugate. This is not obvious a priori but follows from the equations of motion after noticing that the $\mathcal{K}$ part of $A^{\dagger(1)}$ vanishes and that the prepotential for $\bar\psi^{(1)}_q$ is $-iC^{\alpha\beta}\psi^{(0)}_\beta$ (after replacing $q \to q^\dagger$).

Finally the solution for the spinor $\psi^{(1)}$ is

$$\begin{aligned}
\psi^{(1)\dot a}_{\alpha} &= g^{1/2}\frac{4\sqrt{2}i\rho^2 q_\beta C_\alpha{}^{\beta}}{(x^2+\rho^2)^{5/2}}\left(\bar\zeta^2 x^{\dot a}_{\gamma}\eta^\gamma - \rho^2\eta^2\bar\zeta^{\dot a}\right)\\
&= g^{1/2}\frac{4\sqrt{2}i\rho^2 q_\beta C_\alpha{}^{\beta}}{(x^2+\rho^2)^{5/2}}\left(\bar\zeta^2 - \rho^2\eta^2\right)\bar\xi^{\dot a}\,.
\end{aligned} \tag{6.20}$$

As in the zero-th order case, this expression is obtained by acting with a covariant derivative on the equation and inverting the resulting Laplace operator.

6.3 Second order in C

Finally, we complete the iteration solving the equations at second order in the deformation. They read

$$\begin{aligned}
&\bar{\mathcal{D}}^{\dot\alpha\alpha(0)}\mathcal{D}^{(0)}_{\alpha\dot\alpha}A^{(2)} + igkA^{\dot\alpha\alpha(1)}\mathcal{D}^{(0)}_{\alpha\dot\alpha}A^{(1)} - \frac{g^2}{4}k^2A^{\dot\alpha\alpha(1)}A^{(1)}_{\alpha\dot\alpha}A^{(0)} =\\
&\quad = h\sqrt{2}C^{\alpha\beta}\Big[\mathcal{D}^{(0)}_{\alpha\dot\alpha}(\bar\lambda^{\dot\alpha}\psi^{(1)}_\beta) + \frac{ig}{2}kA^{(1)}_{\alpha\dot\alpha}(\bar\lambda^{\dot\alpha}\psi^{(0)}_\beta)\Big]\\
&\bar{\mathcal{D}}^{\dot\alpha\alpha(0)}\mathcal{D}^{(0)}_{\alpha\dot\alpha}A^{\dagger(2)} - igkA^{\dot\alpha\alpha(1)}\mathcal{D}^{(0)}_{\alpha\dot\alpha}A^{\dagger(1)} - \frac{g^2}{4}k^2A^{\dot\alpha\alpha(1)}A^{(1)}_{\alpha\dot\alpha}A^{\dagger(0)} = -i\sqrt{2}g\bar\psi^{(2)}_{\dot\alpha}\bar\lambda^{\dot\alpha}\\
&\bar{\mathcal{D}}^{\dot\alpha\alpha(0)}\psi^{(2)}_\alpha + \frac{ig}{2}kA^{\dot\alpha\alpha(1)}\psi^{(1)}_\alpha = -\frac{\sqrt{2}}{2}g\bar\lambda^{\dot\alpha}A^{(2)}\\
&\mathcal{D}^{(0)}_{\alpha\dot\alpha}\bar\psi^{\dot\alpha(2)} - \frac{ig}{2}kA^{(1)}_{\alpha\dot\alpha}\bar\psi^{\dot\alpha(1)} = ih\frac{\sqrt{2}}{2}C_\alpha{}^{\beta}\Big[(\mathcal{D}^{(0)}_{\beta\dot\beta}A^{\dagger(1)})\bar\lambda^{\dot\beta} - \frac{ig}{2}kA^{(1)}_{\beta\dot\beta}A^{\dagger(0)}\bar\lambda^{\dot\beta}\Big].
\end{aligned} \tag{6.21}$$

At this stage many simplifications take place as one can easily check by Grassmann counting. For example, the third equation drops out because $A^{\dot\alpha\alpha(1)}\psi^{(1)}_\alpha$ and $\bar\lambda^{\dot\alpha}A^{(2)}$ are of order 5 in $\bar\zeta$ and η and therefore vanish. One would then be left with the homogeneous equation which has no solution in the given instanton background.

Using the same procedure outlined in the previous sections one can solve (6.21) completely. Details of the computations are contained in the appendix B.

The solution for $A^{(2)}$ is quartic in the supersymmetry collective coordinates and reads

$$A^{(2)\dot a} = \frac{C^2 q^\alpha x_\alpha^{\dot a}\bar\zeta^2\eta^2}{(x^2+\rho^2)^{5/2}}\left(\rho^2 - 6x^2\right), \tag{6.22}$$

where we have defined $C^2 = C^{\alpha\beta}C_{\alpha\beta} = 2\det C$.

Using the usual Ansatz (6.16) one can solve the fourth equation for $\bar\psi^{(2)}$. It consists of two contributions

$$\bar\psi^{(2)} = \bar\psi^{(2)}_{\mathcal{K}} + \bar\psi^{(2)}_q, \tag{6.23}$$

where

$$(\bar\psi^{(2)}_{\mathcal{K}})_{\dot\alpha\dot a} = g^{-1/2}\frac{4\mathcal{K}C^2\bar\zeta^2\eta^2\epsilon_{\dot\alpha\dot a}}{3(x^2+\rho^2)^{7/2}}\left(x^4+2x^2\rho^2-5\rho^4\right)$$
$$(\bar\psi^{(2)}_q)_{\dot\alpha\dot a} = g^{-1/2}\frac{2\sqrt{2}hq^\dagger_\gamma C^2\epsilon_{\dot\alpha\dot a}}{(x^2+\rho^2)^{7/2}}\left[\bar\zeta^2\eta^\gamma(\rho^4+2x^2\rho^2-x^4)+2\rho^4\eta^2\bar\zeta^{\dot\gamma}x^\gamma_{\dot\gamma}\right]. \tag{6.24}$$

Finally, the solution for $A^{\dagger(2)}$ is

$$A^{\dagger(2)}_{\dot a} = -\frac{C^2 q^\dagger_\alpha x^\alpha_{\dot a}\bar\zeta^2\eta^2}{(x^2+\rho^2)^{5/2}}\left(2\rho^2+6x^2\right). \tag{6.25}$$

This solution goes as $1/x^2$ and therefore will contribute to the effective action presented in section 7. The part of the solution proportional to $\mathcal{K}$ drops out because it contains too many powers of $\bar\zeta$ and η.

It is easy to check that there are no contributions of order C^3 because of the grassmannian nature of the collective coordinates which enter in the equations.

7. The effective action

At this point one can substitute the solutions to the equations of motion into the classical action and keep the leading terms. This gives the leading order correction in the coupling constant to the classical gauge instanton action. One can write

$$S_{eff} = \frac{8\pi^2}{g^2} + g^0 S_0^{\rm tot}, \tag{7.1}$$

where ${S_0}^{tot}$ is the sum of the contributions from untilded fields, S_0, and from tilded fields, $\tilde S_0$. Upon partial integration and use of the equations of motion (4.2), one has

$$S_0 = \frac{1}{2}\int d^4x\ \partial^{\dot\alpha\alpha}\left(\left(\mathcal{D}_{\alpha\dot\alpha}A^\dagger\right)A\right) \tag{7.2}$$

and a similar expression for $\tilde S_0$.

Using Gauss's theorem, (7.2) can be converted into a surface integral on the sphere at infinity. Then, one needs to keep only terms of order $1/x^3$. Analyzing the asymptotics of the different fields one is left with the following

$$S_0 \;=\; \frac{1}{2}\mathrm{vol}(S^3)\;\frac{x^{\dot\alpha\alpha}}{|x|}\left(\mathcal{D}^{(0)}_{\alpha\dot\alpha}\left(A^{\dagger(0)}+A^{\dagger(2)}\right)\right)_{\dot\alpha}A^{(0)\dot\alpha}\Big|_{|x|\to\infty}. \tag{7.3}$$

Note that there are no contributions either from $A^{\dagger(1)}$ or the U(1) part of the covariant derivative. As a consequence the effective action does not contain terms linear in the deformation parameter C and is Lorentz invariant.[11] After plugging in the explicit solutions (7.3) becomes

$$S_0 \;=\; 2\pi^2|q|^2\rho^2+4\pi^2\sqrt{2}\rho^2 q^\alpha\eta_\alpha\mathcal{K}+24\pi^2|q|^2C^2\bar\zeta^2\eta^2\,. \tag{7.4}$$

Analogously, $\tilde S_0$ is

$$\tilde S_0 \;=\; 2\pi^2|\tilde q|^2\rho^2+4\pi^2\sqrt{2}\rho^2\tilde{\mathcal{K}}\eta^\alpha\tilde q_\alpha+24\pi^2|\tilde q|^2C^2\bar\zeta^2\eta^2\,. \tag{7.5}$$

Using the D-term constraint (5.12) one has $q^\alpha\eta_\alpha=q\eta_1$ and $\eta^\alpha\tilde q_\alpha=-\tilde q\eta_2$. One can also identify $\mathcal{K}=q^\dagger_\alpha\mathcal{K}^\alpha=q^\dagger\mathcal{K}^1$ and $\tilde{\mathcal{K}}=\mathcal{K}_\alpha\tilde q^{\dagger\alpha}=\tilde q^\dagger\mathcal{K}^2$. This, along with the condition $|q|^2=|\tilde q|^2$, allows one to write the total effective action in the form

$$S_0^{\,tot}=4\pi^2|q|^2\rho^2+4\pi^2\sqrt{2}|q|^2\rho^2\eta_\alpha\mathcal{K}^\alpha+48\pi^2|q|^2C^2\bar\zeta^2\eta^2\,. \tag{7.6}$$

We finally notice that, defining a "deformed instanton size"

$$\rho_C^2\equiv\rho^2+12C^2\bar\zeta^2\eta^2\,, \tag{7.7}$$

the action (7.6) can be recast in a way which is formally equivalent to the undeformed case

$$\begin{aligned} S_0^{\rm tot} \;&=\; 4\pi^2|q|^2\rho_C^2+4\pi^2\sqrt{2}|q|^2\rho_C^2\eta_\alpha\mathcal{K}^\alpha \\ &\equiv\; 4\pi^2|q|^2(\rho_C^{\rm inv})^2\,, \end{aligned} \tag{7.8}$$

where $(\rho_C^{\rm inv})^2\equiv\rho_C^2(1+\sqrt{2}\eta_\alpha\mathcal{K}^\alpha)$. This combination will prove to be invariant under the $\mathcal{N}=1/2$ transformations of the moduli we present in the next section.

8. Supersymmetry transformations of the moduli

In order to check the supersymmetry invariance of the super-instanton effective action at leading order (7.6) we need to know how the different (pseudo)-collective coordinates transform under $\mathcal{N}=1/2$ supersymmetry. To do this we only need to look at the undeformed part of the solution, and equate active $\mathcal{N}=1/2$ supersymmetry transformations acting on the fields to passive transformations acting on the supersymmetric collective coordinates. In this way performing an $\mathcal{N}=1/2$ supersymmetry variation on the background yields another background of the same functional form, but with shifted collective coordinates.

[11] In fact the final result will only depend on the scalar $C^2=2\det C$. This seems to be a general feature. For example, in [17] it was shown that the non-anticommutative deformation of the chiral effective superpotential is also Lorentz invariant, even though the microscopic lagrangian is not.

This is the usual procedure, and we find

$$\begin{aligned}
\delta_\epsilon (x_0)_{\alpha\dot\alpha} &= 4i\epsilon_\alpha \bar\zeta_{\dot\alpha} \\
\delta_\epsilon \rho^2 &= 4i\rho^2 \epsilon^\alpha \eta_\alpha \\
\delta_\epsilon \bar\zeta^{\dot\alpha} &= 0 \\
\delta_\epsilon \eta_\alpha &= 4i\eta_\alpha \epsilon^\beta \eta_\beta \\
\delta_\epsilon \mathcal{K}_\alpha &= 2\sqrt{2} i\epsilon_\alpha - 8i\eta^\beta \mathcal{K}_\beta \epsilon_\alpha \,,
\end{aligned} \tag{8.1}$$

which are the known transformations of the moduli for $\mathcal{N} = 1$ supersymmetric Yang-Mills coupled to matter.

It is straightforward to check that the effective super-instanton action (7.6) is invariant under these transformations. This occurs because of the delicate interplay between the U(1) part of the connection (proportional to the parameter k introduced before) and the Yukawa-like term (proportional to h). In fact, if one keeps h and k arbitrary one would get the following additional contributions to the effective action

$$S_0^{\text{extra}} = 4\pi^2(h-k) q^\dagger_\alpha C^\alpha{}_\beta q^\beta (\bar\zeta^2 - \rho^2\eta^2) + 4\sqrt{2}\pi^2 (h-k)\mathcal{K}\bar\zeta^2 \eta_\alpha C^\alpha{}_\beta q^\beta \tag{8.2}$$

with a similar contribution for the tilded fields. We stress that the second term in (8.2) explicitly breaks the supersymmetry.

Moreover, the usual transformations of the collective coordinates under the antichiral supersymmetries are

$$\begin{aligned}
\delta_{\bar\epsilon} (x_0)_{\alpha\dot\alpha} &= 0 \\
\delta_{\bar\epsilon} \rho^2 &= 0 \\
\delta_{\bar\epsilon} \bar\zeta^{\dot\alpha} &= \bar\epsilon^{\dot\alpha} \\
\delta_{\bar\epsilon} \eta &= 0 \\
\delta_{\bar\epsilon} \mathcal{K}_\alpha &= 0 \,.
\end{aligned} \tag{8.3}$$

We see that the first two terms in (7.6), which come from $\mathcal{N} = 1$ supersymmetric Yang-Mills are invariant under (8.3), but the third one is not. This is at it should, and the explicit breaking of $\mathcal{N} = 1$ to $\mathcal{N} = 1/2$ is reflected at the level of the super-instanton effective action.

9. Gluino condensate

In this final section we comment on how the superspace deformation affects the gluino condensate. We begin by reviewing the computation of the condensate in the instanton background with matter in the undeformed case. This method is usually referred to as the weak coupling approach [18 – 20, 12].

The fundamental idea is that adding matter to pure gluodynamics, and assuming $\langle A\rangle \gg \Lambda_{\mathrm{QCD}}$ allows one to do the calculation consistently in the weakly coupled region. This way we prevent the running of the gauge coupling into the strong coupling region

JHEP10(2005)021

where semi-classical instanton techniques are not reliable. One can then eventually decouple matter and go back to pure gluodynamics using standard renormalization group and holomorphy arguments. In the weak coupling approach we find the gluino condensate directly by integrating the gauge invariant operator $\mathrm{Tr}\bar\lambda\bar\lambda$ over the instanton moduli space $\mathcal{M}$

$$\int_{\mathcal{M}} d\mu\, \mathrm{Tr}\bar\lambda\bar\lambda(x)\,, \tag{9.1}$$

where $d\mu$ is the instanton measure [16]

$$d\mu \sim \frac{M_{\mathrm{PV}}^5}{q^2}\left(\frac{8\pi^2}{g^2}\right)^2 e^{-S_{eff}}\, \frac{d\rho^2}{\rho^2}\, d^4x_0\, d^2\bar\zeta\, d^2\eta\, d^2\mathcal{K}\,, \tag{9.2}$$

where M_{PV} is the Pauli-Villars regularization mass. Note in particular the Grassmann integration over the fundamental collective coordinates. Since the effective action does not contain the $\bar\zeta$ collective coordinate corresponding to the broken susy generator $\bar Q$, the only way to saturate the $d^2\bar\zeta$ part of the measure is to pick the $\bar\zeta^2$ component of the gluino insertion. The Grassmann integral over the remaining collective coordinates is saturated by pulling down powers of the Yukawa term $\eta^\alpha\mathcal{K}_\alpha$ in the 't Hooft effective action S_{eff}. The final result does not depend on the location of the insertion. This follows directly also from the fact $\bar\lambda\bar\lambda$ is the lowest component of an anti-chiral superfield.

Now we turn to the $\mathcal{N}=1/2$ case. The measure (9.2) is altered by the C-dependent contributions coming from[12]

$$\begin{aligned}\left|\left|\frac{\partial\bar\psi}{\partial\mathcal{K}}\right|\right|\left|\left|\frac{\partial\bar{\tilde\psi}}{\partial\tilde{\mathcal{K}}}\right|\right| &\equiv \left(\int d^4x\left|\frac{\partial\bar\psi}{\partial\mathcal{K}^\alpha}\right|^2\right)^{1/2}\left(\int d^4x\left|\frac{\partial\bar{\tilde\psi}}{\partial\mathcal{K}^\alpha}\right|^2\right)^{1/2}\\ &= \pi^2|q|^2\left(\rho^2+\frac{4}{3}(3h^2-k^2)C^2\bar\zeta^2\eta^2\right).\end{aligned} \tag{9.3}$$

For $h=1=k$ this is

$$\left|\left|\frac{\partial\bar\psi}{\partial\mathcal{K}}\right|\right|\left|\left|\frac{\partial\bar{\tilde\psi}}{\partial\tilde{\mathcal{K}}}\right|\right| = \pi^2|q|^2\left(\rho^2+\frac{8}{3}C^2\bar\zeta^2\eta^2\right). \tag{9.4}$$

This enters in the denominator of the measure [16]. The only contributions to $||\frac{\partial\bar\psi}{\partial\mathcal{K}}||$ come from $\left|\left|\frac{\partial\bar\psi^{(0)}}{\partial\mathcal{K}}\right|\right|$ and $||\frac{\partial\bar\psi^{(1)}}{\partial\mathcal{K}}||$. It turns out that $\bar\psi^{(0)}$, $\bar\psi^{(1)}$, and $\bar\psi^{(2)}$ are mutually orthogonal. The undeformed part $||\frac{\partial\bar\psi^{(0)}}{\partial\mathcal{K}}||||\frac{\partial\bar{\tilde\psi}^{(0)}}{\partial\mathcal{K}}||$ gives the usual term $\pi^2|q|^2\rho^2$. From (9.4) and (7.7) one notices that the deformation does not convert $\frac{d\rho^2}{\rho^2}$ into $\frac{d\rho_C^2}{\rho_C^2}$, as one might have expected.

As it is well-known, in the absence of the deformation, the integration of (9.2) (upon substitution of q with a scalar superfield Φ) yields the Affleck-Dine-Seiberg superpotential [18]. After turning on the deformation, one can see that the additional contributions to the effective action and the measure do not modify the superpotential. Indeed, integrating over the fermionic coordinates $\mathcal{K}$ and η first, the C-dependent pieces vanish.

[12] We thank Arkady Vainshtein for suggesting us to investigate this point.

Even though the presence of the deformation modifies the effective action and the measure, it is easy to verify that

$$\int_{\mathcal{M}} d\mu_C\, e^{-48\pi^2|q|^2 C^2 \bar{\zeta}^2 \eta^2} \mathrm{Tr}\bar{\lambda}\bar{\lambda}(x) \tag{9.5}$$

does not depend on C. Indeed the only way to saturate the integral over the fundamental fermionic collective coordinate $\mathcal{K}^\alpha$ is through the term $\mathcal{K}^\alpha \eta_\alpha$ in the undeformed effective action $S_{eff}^{C=0}$. Doing so we saturate also the $d^2\eta$ associated to the broken superconformal transformations moduli. There is no space left then for $C^2\bar{\zeta}^2\eta^2$. We conclude that the gluino condensate is not modified.

We can support the above conclusion also through the following formal argument. It was noted in [8] that varying the pure SYM lagrangian with respect to the deformation parameter C yields a Q_α exact term

$$\delta_C \mathcal{L}_{\mathrm{gauge}} = \delta C^{\mu\nu} \left(i F_{\mu\nu} \bar{\lambda}\bar{\lambda} - \frac{C_{\mu\nu}}{4} (\bar{\lambda}\bar{\lambda})^2 \right) \sim \delta C^{\mu\nu} \{ Q^\alpha, (\sigma_{\mu\nu})_{\alpha\beta} \lambda^\beta \bar{\lambda}\bar{\lambda} \}. \tag{9.6}$$

The new thing in the present context is the C variation of the matter sector. After setting the auxiliary fields to zero this reads

$$\delta_C \mathcal{L}_{\mathrm{mat}} = -\frac{\sqrt{2}}{2} \delta C^{\alpha\beta} \mathcal{D}_{\alpha\dot{\alpha}} A^\dagger \bar{\lambda}^{\dot{\alpha}} \psi_\beta - \frac{\sqrt{2}}{2} \delta C^{\alpha\beta} \tilde{\psi}_\beta \bar{\lambda}^{\dot{\alpha}} \mathcal{D}_{\alpha\dot{\alpha}} \tilde{A}^\dagger \ \sim \ \delta C^{\alpha\beta} \{ Q_\alpha, \bar{\psi}\bar{\lambda}\psi_\beta + \tilde{\psi}_\beta \bar{\lambda} \bar{\tilde{\psi}} \}\,. \tag{9.7}$$

Therefore the matter sector is also Q-exact under the C-deformation. Let us now apply this argument to discuss the dependence of the gluino condensate $\langle \bar{\lambda}\bar{\lambda} \rangle$ on the deformation parameter. The C variation of the action inserts a $\delta_{Q_\alpha}(\ \cdot\ )$ in the correlation function. The δ_{Q_α} operator can be pulled past the insertion to act on the vacuum. Therefore we conclude that

$$\frac{\delta}{\delta C_{\mu\nu}} \langle \bar{\lambda}\bar{\lambda}(x) \rangle = 0 \tag{9.8}$$

and we recover the familiar $\mathcal{N}=1$ result. We cannot make similar considerations for antichiral insertions of $A^\dagger$ because the profile now depends explicitly on C. The C deformation can give non trivial contributions to correlation functions like $\langle A^\dagger \tilde{A}^\dagger \rangle$ and $\langle \bar{\psi} \bar{\tilde{\psi}} \rangle$.

10. Conclusion

In this paper we analyzed the effect of the $\mathcal{N}=1/2$ superspace deformation on the instanton calculus in presence of a matter superfield. The field equations have been solved iteratively in the deformation parameter and at leading order in the coupling constant. The fermionic nature of the back-reaction allows to find the exact C dependence of the solution. The matter part receives linear and quadratic corrections in C. The asymptotic behavior of the C-deformed matter solution modifies the 't Hooft supersymmetric effective action. The correction is quadratic in C, quartic in the collective coordinates and $\mathcal{N}=1/2$ supersymmetric. The modified effective action does not alter the gluino condensate.

JHEP10(2005)021

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A. Notation and conventions

In this appendix we define our notation and conventions and collect some useful formulas.

Spinor algebra. We work in euclidean space R^4 but we continue to adopt the lorenztian signature notation. The spinor notation is based on the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ algebra of the Lorentz group. In Euclidean space the SU(2) subalgebras are not related by complex conjugation.

The spinor indices $\alpha, \dot{\alpha} = 1, 2$ are raised and lowered with the ϵ tensor in the following way

$$\begin{aligned} \psi^\alpha &= \epsilon^{\alpha\beta}\psi_\beta & \qquad \psi_\alpha &= \epsilon_{\alpha\beta}\psi^\beta \\ \bar{\psi}^{\dot{\alpha}} &= \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}} & \qquad \bar{\psi}_{\dot{\alpha}} &= \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}} \,. \end{aligned} \tag{A.1}$$

Useful identities involving ϵ tensors are

$$\begin{aligned} \epsilon_{\alpha\beta}\epsilon^{\beta\gamma} &= \delta_\alpha^\gamma & \qquad \epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\beta}\dot{\gamma}} &= \delta_{\dot{\alpha}}^{\dot{\gamma}} \\ \epsilon_{\alpha\beta}\epsilon^{\delta\gamma} &= \delta_\alpha^\gamma\delta_\beta^\delta - \delta_\alpha^\delta\delta_\beta^\gamma & \qquad \epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\delta}\dot{\gamma}} &= \delta_{\dot{\alpha}}^{\dot{\gamma}}\delta_{\dot{\beta}}^{\dot{\delta}} - \delta_{\dot{\alpha}}^{\dot{\delta}}\delta_{\dot{\beta}}^{\dot{\gamma}} \,. \end{aligned} \tag{A.2}$$

The spinors are contracted according to the following rules

$$\begin{aligned} \psi\chi &= \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi \\ \bar{\psi}\bar{\chi} &= \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = -\bar{\psi}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = \bar{\chi}\bar{\psi} \end{aligned} \tag{A.3}$$

and

$$\begin{aligned} \psi^\alpha\psi^\beta &= -\frac{1}{2}\epsilon^{\alpha\beta}\psi\psi & \qquad \psi_\alpha\psi_\beta &= \frac{1}{2}\epsilon_{\alpha\beta}\psi\psi \\ \bar{\psi}^{\dot{\alpha}}\bar{\psi}^{\dot{\beta}} &= \frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}\bar{\psi} & \qquad \bar{\psi}_{\dot{\alpha}}\bar{\psi}_{\dot{\beta}} &= -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}\bar{\psi} \,. \end{aligned} \tag{A.4}$$

We adopt the following definition

$$x^2 \equiv \frac{1}{2}x^{\dot{\alpha}\alpha}x_{\alpha\dot{\alpha}} = -x^\mu x_\mu \tag{A.5}$$

from which follows that

$$x_{\alpha\dot{\beta}}x^{\dot{\beta}a} = \delta_\alpha^a x^2 \,. \tag{A.6}$$

The derivative acts as

$$\partial_{\alpha\dot{\alpha}}x^{\beta\dot{\beta}} = -2\delta_\alpha^\beta\delta_{\dot{\alpha}}^{\dot{\beta}} \,. \tag{A.7}$$

Covariant derivative. The covariant derivative with the undeformed connection is

$$\mathcal{D}^{(0)\dot{a}}_{\alpha\dot{\alpha}\ \dot{b}} = \partial_{\alpha\dot{\alpha}}\delta^{\dot{a}}_{\dot{b}} \pm \frac{1}{x^2+\rho^2}(\delta^{\dot{a}}_{\dot{\alpha}}x_{\alpha\dot{b}} + \epsilon_{\dot{b}\dot{\alpha}}x^{\dot{a}}_{\alpha})\,. \tag{A.8}$$

The $+$ sign is used when the derivative acts on fundamental fields and the $-$ sign when it acts on antifundamental ones.

Propagator. In order to solve Poisson equations in the SU(2) self-dual instanton background we repeatedly use the following propagator which is the inverse of $-(\mathcal{D}^{(0)})^2$ [15]

$$G^{\dot{a}}_{\dot{b}}(x,y) = \frac{i}{4\pi^2}\frac{x^{\dot{a}\gamma}y_{\gamma\dot{b}} + \rho^2\delta^{\dot{a}}_{\dot{b}}}{(x^2+\rho^2)^{1/2}(y^2+\rho^2)^{1/2}(x-y)^2}\,. \tag{A.9}$$

B. Solutions order by order in C

In this appendix we give details about the procedure for getting the solutions to the matter field equations order by order in C. In particular we present explicit expressions for the currents appearing in the Poisson equations.

B.1 Zero-th order in C

The solution for $A^{\dagger(0)}$ is given by solving

$$\left(\mathcal{D}^{(0)2}A^{\dagger(0)}\right)_{\dot{a}} = i\frac{\sqrt{2}}{2}g\bar{\psi}^{(0)}_{\dot{\alpha}\dot{b}}\bar{\lambda}^{\dot{\alpha}\dot{b}}{}_{\dot{a}} = -\frac{12\sqrt{2}\mathcal{K}\rho^4}{(x^2+\rho^2)^{7/2}}\bar{\xi}_{\dot{a}} \equiv \mathcal{J}_{(A^\dagger)\dot{a}}\,. \tag{B.1}$$

The operator $(\mathcal{D}^{(0)})^2$ can be inverted using (A.9). The convolution

$$A^{\dagger(0)}_{\dot{a}}(x) = -\int d^4y\mathcal{J}_{(A^\dagger)\dot{b}}(y)G^{\dot{b}}{}_{\dot{a}}(y,x) \tag{B.2}$$

yields a particular solution. Adding the homogeneuos solution (which is, up to the position of the indices, the same as for $A^{(0)}$) one obtains

$$A^{\dagger(0)}_{\dot{a}}(x,\rho,\mathcal{K},\bar{\xi}) = q^\dagger_\alpha\frac{x^\alpha_{\dot{a}}}{\sqrt{x^2+\rho^2}} - \frac{\sqrt{2}\mathcal{K}\rho^2\bar{\xi}_{\dot{a}}}{(x^2+\rho^2)^{3/2}}\,. \tag{B.3}$$

The solution for $\psi^{(0)}$ is given by acting with a covariant derivative on both sides of its equation in (6.5) and by solving

$$\left(\mathcal{D}^{(0)2}\psi^{(0)}_\alpha\right)^{\dot{a}} = \frac{\sqrt{2}}{2}g\mathcal{D}^{(0)}_{\alpha\dot{\alpha}}(\bar{\lambda}^{\dot{\alpha}\dot{a}}{}_{\dot{b}}A^{(0)\dot{b}}) \equiv \mathcal{J}^{\dot{a}}_{(\psi)\alpha}\,, \tag{B.4}$$

where the current is

$$\mathcal{J}^{\alpha\dot{a}}_{(\psi)} = g^{1/2}\frac{4\sqrt{2}iq^\beta\rho^2}{(x^2+\rho^2)^{7/2}}\left[\delta^\alpha_\beta\bar{\xi}^{\dot{a}}(x^2+6\rho^2) + x^{\dot{a}}_\beta x^\alpha_{\dot{c}}\bar{\xi}^{\dot{c}} - x^{\dot{a}\alpha}x_{\beta\dot{c}}\bar{\xi}^{\dot{c}}\right] = g^{1/2}\frac{24\sqrt{2}iq^\alpha\rho^4}{(x^2+\rho^2)^{7/2}}\bar{\xi}^{\dot{a}}\,. \tag{B.5}$$

The last line is obtained by using the following identity

$$x^{\dot{a}}_\alpha x^{\dot{b}}_\beta - x^{\dot{b}}_\alpha x^{\dot{a}}_\beta = -\epsilon_{\alpha\beta}\epsilon^{\dot{a}\dot{b}}x^2\,. \tag{B.6}$$

By convoluting with the propagator in (A.9) one gets (in this case the homogeneous solution is vanishing)

$$\psi^{\alpha\dot{a}(0)}(x,\rho,\bar{\xi}) = -g^{1/2}\frac{2\sqrt{2}iq^\alpha\rho^2}{(x^2+\rho^2)^{3/2}}\bar{\xi}^{\dot{a}}\,. \tag{B.7}$$

JHEP10(2005)021

B.2 First order in C

One starts by plugging in the first equation in (6.14) the explicit solutions for the fields at zero-th order in C. After taking all the derivatives one can rewrite the equation explicitly as

$$\begin{aligned}\left(\mathcal{D}^{(0)2}A^{(1)}\right)^{\dot{a}} &= -\frac{16k\rho^2 q_\alpha C^{\alpha\beta}}{(x^2+\rho^2)^{9/2}}\Big[x^{\dot{a}}_{\beta}(x^2+3\rho^2)\bar{\zeta}^2 - 2x^{\dot{a}}_{\beta}x^2\rho^2\eta^2 - \\ &\qquad - 2\rho^2\left(2x^{\dot{a}}_{\beta}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma - (x^2+\rho^2)\bar{\zeta}^{\dot{a}}\eta_\beta\right)\Big] + \\ &\quad + \frac{48h\rho^4 q_\alpha C^{\alpha\beta}}{(x^2+\rho^2)^{9/2}}\Big[2x^{\dot{a}}_{\beta}\bar{\zeta}^2 + x^{\dot{a}}_{\beta}\eta^2(\rho^2-x^2) - \\ &\qquad - 2\left(2x^{\dot{a}}_{\beta}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma - (x^2+\rho^2)\bar{\zeta}^{\dot{a}}\eta_\beta\right)\Big]. \end{aligned} \tag{B.8}$$

Using $\bar{\mathcal{D}}^{\dot{\alpha}\alpha(0)}\mathcal{D}^{(0)}_{\alpha\dot{\alpha}} = -2(\mathcal{D}^{(0)})^2$ and taking the convolution with (A.9) one gets

$$\begin{aligned}A^{(1)\dot{a}} &= \frac{q_\alpha C^{\alpha\beta}}{(x^2+\rho^2)^{5/2}}\Big[x^{\dot{a}}_{\beta}\left(2(h-k)x^2+(5h-3k)\rho^2\right)\bar{\zeta}^2 + \\ &\quad + x^{\dot{a}}_{\beta}\left(2(k-h)x^2+(h+k)\rho^2\right)\rho^2\eta^2 + \\ &\quad + 2\rho^2(k-3h)\left(x^{\dot{a}}_{\beta}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma - (x^2+\rho^2)\bar{\zeta}^{\dot{a}}\eta_\beta\right)\Big]. \end{aligned} \tag{B.9}$$

By putting $k=1$ and $h=1$ this result becomes (6.15).

The equation for $\bar{\psi}^{(1)}$ in (6.14) can be rewritten in a more manageable way[13] using the Ansatz already discussed in the main text

$$\bar{\psi}^{\dot{\alpha}(1)} = \bar{\mathcal{D}}^{\dot{\alpha}\beta}\Psi^{(1)}_{\beta}. \tag{B.12}$$

The equation then becomes

$$(\mathcal{D}^{(0)})^2\Psi^{(1)}_{\alpha} = -\frac{ig}{2}kA^{(1)}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}(0)} - ih\frac{\sqrt{2}}{2}C_{\alpha}^{\ \beta}(\mathcal{D}^{(0)}_{\beta\dot{\beta}}A^{\dagger(0)})\bar{\lambda}^{\dot{\beta}} \equiv \mathcal{J}^{(\bar{\psi}^{(1)})}_{\alpha}. \tag{B.13}$$

The explicit expression for the current reads

$$\begin{aligned}\mathcal{J}^{(\bar{\psi}^{(1)})}_{\alpha\dot{a}} &= -\frac{8k\rho^2 C_{\alpha}^{\ \beta}\mathcal{K}}{(x^2+\rho^2)^{9/2}}\Big[x_{\beta\dot{a}}(x^2+3\rho^2)\bar{\zeta}^2 - 2x_{\beta\dot{a}}x^2\rho^2\eta^2 - \\ &\qquad - 2\rho^2\left(2x_{\beta\dot{a}}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma - (x^2+\rho^2)\bar{\zeta}_{\dot{a}}\eta_\beta\right)\Big] + \end{aligned}$$

[13] Using the fact that the U(1) part of the connection can be rewritten as

$$A^{(1)}_{\alpha\dot{\alpha}} = -C_{\alpha}^{\ \beta}\partial_{\beta\dot{\alpha}}K = -C_{\alpha}^{\ \beta}\mathcal{D}^{(0)}_{\beta\dot{\alpha}}K \tag{B.10}$$

one could think to rewrite this equation as a total covariant derivative

$$\mathcal{D}^{\beta(0)}_{\dot{\alpha}}\left[\epsilon_{\alpha\beta}\bar{\psi}^{\dot{\alpha}(1)} + \frac{iC_{\alpha\beta}}{2}(K\bar{\psi}^{\dot{\alpha}(0)} - 2\sqrt{2}A^{\dagger(0)}\bar{\lambda}^{\bar{\alpha}})\right] = 0 \tag{B.11}$$

and then obtain a solution by putting to zero the argument of the derivative. Actually, this does not yield a solution since $C_{\alpha\beta}$ is a symmetric tensor.

$$+\frac{24h\rho^4\mathcal{K}C_\alpha{}^\beta}{(x^2+\rho^2)^{9/2}}\Big[2x_{\beta\dot{a}}\bar{\zeta}^2+x_{\beta\dot{a}}\eta^2(\rho^2-x^2)- \tag{B.14}$$
$$-2\left(2x_{\beta\dot{a}}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma-(x^2+\rho^2)\bar{\zeta}_{\dot{a}}\eta_\beta\right)\Big]--\frac{24\sqrt{2}h\rho^4q^\dagger_\beta C_\alpha{}^\beta}{(x^2+\rho^2)^{7/2}}\bar{\xi}_{\dot{a}}\,.$$

Taking the convolution of (B.14) with (A.9) one obtains the prepotential

$$\bar{\Psi}^{(1)}_{\alpha\dot{a}}=\frac{\mathcal{K}C_\alpha{}^\beta}{2(x^2+\rho^2)^{5/2}}\Big[x_{\beta\dot{a}}\left(2(h-k)x^2+(5h-3k)\rho^2\right)\bar{\zeta}^2+$$
$$+x_{\beta\dot{a}}\left(2(k-h)x^2+(h+k)\rho^2\right)\rho^2\eta^2+$$
$$+2\rho^2(k-3h)\left(x_{\beta\dot{a}}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma-(x^2+\rho^2)\bar{\zeta}_{\dot{a}}\eta_\beta\right)\Big]-$$
$$-\frac{2\sqrt{2}h\rho^2q^\dagger_\beta C_\alpha{}^\beta}{(x^2+\rho^2)^{3/2}}\bar{\xi}_{\dot{a}}\,. \tag{B.15}$$

Acting on this with a covariant derivative we finally obtain

$$\bar{\psi}^{(1)}=\bar{\psi}^{(1)}_{\bar{\zeta}^2}+\bar{\psi}^{(1)}_{\eta^2}+\bar{\psi}^{(1)}_{\bar{\zeta}\eta}+\bar{\psi}^{(1)}_{linear} \tag{B.16}$$

where

$$(\bar{\psi}^{(1)}_{\bar{\zeta}^2})^{\dot{\alpha}}_{\dot{a}}=g^{-1/2}\frac{2\mathcal{K}C_\alpha^{\ \beta}x^{\dot{\alpha}\alpha}x_{\beta\dot{a}}\bar{\zeta}^2}{(x^2+\rho^2)^{7/2}}\left((h-k)x^2+2(2h-k)\rho^2\right)$$
$$(\bar{\psi}^{(1)}_{\eta^2})^{\dot{\alpha}}_{\dot{a}}=g^{-1/2}\frac{2\mathcal{K}C_\alpha^{\ \beta}x^{\dot{\alpha}\alpha}x_{\beta\dot{a}}\rho^2\eta^2}{(x^2+\rho^2)^{7/2}}\left((k-h)x^2+2h\rho^2\right)$$
$$(\bar{\psi}^{(1)}_{\bar{\zeta}\eta})^{\dot{\alpha}}_{\dot{a}}=g^{-1/2}\frac{4\mathcal{K}C_\alpha^{\ \beta}x^{\dot{\alpha}\alpha}\rho^2}{(x^2+\rho^2)^{7/2}}(k-3h)\left(x_{\beta\dot{a}}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma-(x^2+\rho^2)\bar{\zeta}_{\dot{a}}\eta_\beta\right)$$
$$(\bar{\psi}^{(1)}_{\rm linear})^{\dot{\alpha}}_{\dot{a}}=g^{-1/2}\frac{4\sqrt{2}hq^\dagger_\alpha C^{\alpha\beta}\rho^2}{(x^2+\rho^2)^{5/2}}\left(x^{\dot{\alpha}}_\beta\bar{\xi}_{\dot{a}}-\delta^{\dot{\alpha}}_{\dot{a}}(x_{\beta\dot{\beta}}\bar{\zeta}^{\dot{\beta}}+\rho^2\eta_\beta)\right). \tag{B.17}$$

One has (6.17) by setting $k=1$ and $h=1$.

Having solved for $\bar{\psi}^{(1)}$ allows one to tackle the equation for $A^{\dagger(1)}$. Explicitly this equation reads

$$\left(\mathcal{D}^{(0)2}A^{\dagger(1)}\right)_{\dot{a}}=-\frac{16k\rho^2q^\dagger_\alpha C^{\alpha\beta}}{(x^2+\rho^2)^{9/2}}\Big[x_{\beta\dot{a}}(x^2+3\rho^2)\bar{\zeta}^2-2x_{\beta\dot{a}}x^2\rho^2\eta^2-$$
$$-2\rho^2\left(2x_{\beta\dot{a}}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma-(x^2+\rho^2)\bar{\zeta}_{\dot{a}}\eta_\beta\right)\Big]-$$
$$-\frac{48\sqrt{2}k\rho^4\mathcal{K}C^{\alpha\beta}x_{\beta\dot{a}}}{(x^2+\rho^2)^{9/2}}\left(\bar{\zeta}^2\eta_\alpha-x_{\alpha\dot{\beta}}\eta^2\bar{\zeta}^{\dot{\beta}}\right)+$$
$$+\frac{48h\rho^4q^\dagger_\beta C_{\alpha\beta}}{(x^2+\rho^2)^{9/2}}\Big[2x_{\beta\dot{a}}\bar{\zeta}^2+x_{\beta\dot{a}}\eta^2(\rho^2-x^2)-$$
$$-2\left(2x_{\beta\dot{a}}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma-(x^2+\rho^2)\bar{\zeta}_{\dot{a}}\eta_\beta\right)\Big]+$$

$$+\frac{8\sqrt{2}\rho^2\mathcal{K}C^{\alpha\beta}x_{\beta\dot{\alpha}}}{(x^2+\rho^2)^{9/2}}\Big[\left((h-k)x^2+(9h-3k)\rho^2\right)\bar{\zeta}^2\eta_\alpha+$$
$$+(2k-8h)\rho^2 x_{\alpha\dot{\beta}}\eta^2\bar{\zeta}^{\dot{\beta}}\Big]. \tag{B.18}$$

The convolution with (A.9) yields

$$\begin{aligned}A^{\dagger(1)}_{\dot{\alpha}} &= \frac{q^\dagger_\alpha C^{\alpha\beta}}{(x^2+\rho^2)^{5/2}}\Big[x_{\beta\dot{\alpha}}\left(2(h-k)x^2+(5h-3k)\rho^2\right)\bar{\zeta}^2+\\ &\quad + x_{\beta\dot{\alpha}}\left(2(k-h)x^2+(h+k)\rho^2\right)\rho^2\eta^2+\\ &\quad + 2\rho^2(k-3h)\left(x_{\beta\dot{\alpha}}\bar{\zeta}_{\dot{\gamma}}x^{\dot{\gamma}\gamma}\eta_\gamma-(x^2+\rho^2)\bar{\zeta}_{\dot{\alpha}}\eta_\beta\right)\Big]+\\ &\quad + \frac{2\sqrt{2}\mathcal{K}C^{\alpha\beta}x_{\alpha\dot{\alpha}}}{(x^2+\rho^2)^{5/2}}(h-k)\left[(x^2+2\rho^2)\bar{\zeta}^2\eta_\beta-\rho^2\eta^2\bar{\zeta}^{\dot{\beta}}x_{\beta\dot{\beta}}\right].\end{aligned} \tag{B.19}$$

The final solution with $k=1=h$ is given in (6.19).

The equation for $\psi^{(1)}$ can be cast into

$$\begin{aligned}&\left(\mathcal{D}^{(0)2}\psi^{(1)}_\alpha\right)^{\dot{\alpha}} =\\ &= \frac{16i\sqrt{2}\rho^2 q_\gamma C^{\gamma\beta}}{(x^2+\rho^2)^{9/2}}\bar{\zeta}^2\Big[\epsilon_{\alpha\beta}x^{\dot{\alpha}}_\lambda\eta^\lambda\left((h-3k)x^2+(9k-3h)\rho^2\right)+\\ &\quad + x^{\dot{\alpha}}_\alpha\eta_\beta(k-h)\left(x^2+9\rho^2\right)\Big]+\\ &\quad + \frac{8i\sqrt{2}\rho^4 q_\gamma C^{\gamma\beta}}{(x^2+\rho^2)^{9/2}}\eta^2\left[\epsilon_{\alpha\beta}\bar{\zeta}^{\dot{\alpha}}\left((k+5h)x^2-(3h+7k)\rho^2\right)+16(h-k)x^{\dot{\alpha}}_\beta x_{\alpha\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}}\right].\end{aligned} \tag{B.20}$$

Inverting the covariant laplacian we get

$$\psi^{(1)}=\psi^{(1)}_{\eta^2\bar{\zeta}}+\psi^{(1)}_{\bar{\zeta}^2\eta} \tag{B.21}$$

with

$$\begin{aligned}(\psi^{(1)}_{\eta^2\bar{\zeta}})^{\dot{\alpha}}_\alpha &= g^{1/2}\frac{2\sqrt{2}iq_\gamma C^{\gamma\beta}\rho^2\eta^2}{(x^2+\rho^2)^{5/2}}\left[2(h-k)x^{\dot{\alpha}}_\beta x_{\alpha\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}}+\rho^2\epsilon_{\beta\alpha}\bar{\zeta}^{\dot{\alpha}}(k+h)\right]\\ (\psi^{(1)}_{\bar{\zeta}^2\eta})^{\dot{\alpha}}_\alpha &= g^{1/2}\frac{2\sqrt{2}iq_\gamma C^{\gamma\beta}\bar{\zeta}^2}{(x^2+\rho^2)^{5/2}}\left[\rho^2(3k-h)\epsilon_{\alpha\beta}x^{\dot{\alpha}}_\lambda\eta^\lambda+2(k-h)x^{\dot{\alpha}}_\alpha\eta_\beta(x^2+2\rho^2)\right].\end{aligned} \tag{B.22}$$

This yields (6.20) when $k=1=h$.

B.3 Second order in C

The first equation of (6.21) reads explicitly

$$\begin{aligned}\left(\mathcal{D}^{(0)2}A^{(2)}\right)^{\dot{\alpha}} &= \frac{32k^2\rho^4C^2q^\alpha x^{\dot{\alpha}}_\alpha\bar{\zeta}^2\eta^2}{(x^2+\rho^2)^{9/2}}-\frac{64k^2\rho^2C^2q^\alpha x^{\dot{\alpha}}_\alpha\bar{\zeta}^2\eta^2}{(x^2+\rho^2)^{9/2}}(2x^2-\rho^2)-\\ &\quad -\frac{16h\rho^2C^2q^\alpha x^{\dot{\alpha}}_\alpha\bar{\zeta}^2\eta^2}{(x^2+\rho^2)^{9/2}}\left((h-k)x^2+3(h+k)\rho^2\right)+\\ &\quad +\frac{96hk\rho^4C^2q^\alpha x^{\dot{\alpha}}_\alpha\bar{\zeta}^2\eta^2}{(x^2+\rho^2)^{9/2}},\end{aligned} \tag{B.23}$$

where we have defined $C^2 = C^{\alpha\beta}C_{\alpha\beta}$, and therefore $C_{\alpha\beta}C^{\beta\gamma} = \frac{1}{2}\delta_\alpha^\gamma C^2$. The solution to this equation is

$$A^{(2)\dot{\alpha}} = \frac{C^2 q^\alpha x_\alpha^{\dot{\alpha}} \bar{\zeta}^2 \eta^2}{(x^2+\rho^2)^{5/2}} \left((-6k^2+2hk-2h^2)x^2 + (k^2+3hk-3h^2)\rho^2\right). \tag{B.24}$$

The final result with $k = 1 = h$ is reported in the main text (6.22).

The equation for $\bar{\psi}^{(2)}$ is again easily rewritten using the Ansatz (B.12). In terms of the prepotential we then obtain

$$\begin{aligned}
&\left(\mathcal{D}^{(0)2}\bar{\Psi}^{(2)}_\alpha\right)_{\dot{\alpha}} = \\
&= g^{-1/2}\frac{16\sqrt{2}h\rho^2 q_\gamma^\dagger}{(x^2+\rho^2)^{9/2}}\bar{\zeta}^2\Big[C^2\delta_\alpha^\gamma x_{\delta\dot{\alpha}}\eta^\delta\left((h-k)x^2+3h\rho^2\right) - C^2\eta^\gamma k x_{\alpha\dot{\alpha}}x^2 + \\
&\qquad + C_{\alpha\beta}C^{\gamma\delta}\left(6k\rho^2\eta_\delta x_{\dot{\alpha}}^\beta - \eta^\beta x_{\delta\dot{\alpha}}\left((h-k)x^2+(9h-3k)\rho^2\right)\right)\Big] + \\
&\quad + g^{-1/2}\frac{4\sqrt{2}h\rho^4 q_\gamma^\dagger}{(x^2+\rho^2)^{9/2}}\eta^2\Big[C^2\delta_\alpha^\gamma\bar{\zeta}_{\dot{\alpha}}\left((5h+5k)x^2+(k-3h)\rho^2\right) + 4kC^2\bar{\zeta}_{\dot{\gamma}}x_{\alpha\dot{\alpha}}x^{\dot{\gamma}\gamma} - \\
&\qquad - 8C_{\alpha\beta}C^{\gamma\delta}\left(3k\bar{\zeta}^{\dot{\gamma}}x_{\delta\dot{\gamma}}x_{\dot{\alpha}}^\beta + \bar{\zeta}^{\dot{\gamma}}x_{\delta\dot{\alpha}}x_{\dot{\gamma}}^\beta(k-4h)\right)\Big] + \\
&\quad + g^{-1/2}\frac{8\rho^2\mathcal{K}C^2\bar{\zeta}^2\eta^2 x_{\alpha\dot{\alpha}}}{(x^2+\rho^2)^{9/2}}\left((3k-h)(h-k)x^2+(3h^2+k^2)\rho^2\right).
\end{aligned} \tag{B.25}$$

Inverting the laplacian yields

$$\begin{aligned}
&\bar{\Psi}^{(2)\alpha}_{\dot{\alpha}} = \\
&= g^{-1/2}\frac{\sqrt{2}hq_\gamma^\dagger}{2(x^2+\rho^2)^{5/2}}\bar{\zeta}^2\Big[2C^2\epsilon^{\alpha\gamma}\eta^\beta x_{\beta\dot{\alpha}}(h-k)\left(2x^2+3\rho^2\right) - \\
&\qquad - C^2k\eta^\gamma x_{\dot{\alpha}}^\alpha(2x^2+\rho^2) + 8C^\alpha{}_\beta C^{\gamma\delta}x_{\delta\dot{\alpha}}\eta^\beta(k-h)(x^2+2\rho^2)\Big] + \\
&\quad + g^{-1/2}\frac{\sqrt{2}h\rho^2 q_\gamma^\dagger}{2(x^2+\rho^2)^{5/2}}\eta^2\Big[\rho^2C^2\epsilon^{\alpha\gamma}\epsilon_{\dot{\alpha}\dot{\gamma}}\bar{\zeta}^{\dot{\gamma}}(k-2h) - \\
&\qquad - kC^2\bar{\zeta}^{\dot{\gamma}}x_{\dot{\gamma}}^\alpha x_{\dot{\alpha}}^\gamma + C^{\alpha\beta}C^{\gamma\delta}\bar{\zeta}^{\dot{\gamma}}(8k-8h)x_{\delta\dot{\alpha}}x_{\beta\dot{\gamma}}\Big] + \\
&+ g^{-1/2}\frac{\mathcal{K}C^2x_{\dot{\alpha}}^\alpha}{3(x^2+\rho^2)^{5/2}}\bar{\zeta}^2\eta^2\left(2k(3h-2k)x^2+(3h^2+3hk-k^2)\rho^2\right).
\end{aligned} \tag{B.26}$$

Finally, the covariant derivative on (B.26) yields the result for $\bar{\psi}^{(2)}$. It consists of three contributions

$$\bar{\psi}^{(2)} = \bar{\psi}^{(2)}_{\bar{\zeta}^2\eta} + \bar{\psi}^{(2)}_{\eta^2\bar{\zeta}} + \bar{\psi}^{(2)}_{\bar{\zeta}^2\eta^2}, \tag{B.27}$$

where

$$\begin{aligned}
(\bar{\psi}^{(2)}_{\bar{\zeta}^2\eta})_{\dot{\alpha}\dot{a}} = g^{-1/2}\frac{\sqrt{2}hq_\gamma^\dagger\bar{\zeta}^2}{(x^2+\rho^2)^{7/2}}\Big[&\eta^\delta C^2 x_{\delta\dot{a}}x_{\dot{\alpha}}^\gamma(4h-4k)(x^2+2\rho^2) - \\
&- \eta^\gamma C^2\epsilon_{\dot{\alpha}\dot{a}}(2kx^4-4kx^2\rho^2-2h\rho^4) +
\end{aligned}$$

$$+\eta^\beta C^\alpha{}_\beta C^{\gamma\delta}x_{\alpha\dot\alpha}x_{\delta\dot a}(8h-8k)(x^2+3\rho^2)\Big]$$

$$(\bar\psi^{(2)}_{\eta^2\bar\zeta})_{\dot\alpha\dot a} = g^{-1/2}\frac{\sqrt{2}hq^\dagger_\gamma\rho^2\eta^2}{(x^2+\rho^2)^{7/2}}\Big[\bar\zeta^{\dot\gamma}\rho^2C^2x^\gamma_{\dot\gamma}\epsilon_{\dot\alpha\dot a}(6k-2h)+$$

$$+\bar\zeta_{\dot a}\rho^2C^2x^\gamma_{\dot\alpha}(4k-4h)+\bar\zeta^{\dot\gamma}C^{\alpha\beta}C^{\gamma\delta}x_{\beta\dot\gamma}x_{\alpha\dot\alpha}x_{\delta\dot a}(16h-16k)\Big]$$

$$(\bar\psi^{(2)}_{\bar\zeta^2\eta^2})_{\dot\alpha\dot a} = g^{-1/2}\frac{4\mathcal{K}C^2\bar\zeta^2\eta^2\epsilon_{\dot\alpha\dot a}}{3(x^2+\rho^2)^{7/2}}\Big[(3hk-2k^2)x^4+(3h^2+5k^2-6hk)x^2\rho^2$$

$$+(k^2-3h^2-3hk)\rho^4\Big]. \tag{B.28}$$

The final result for $k=1=h$ is given in (6.24).

The equation for $A^{\dagger(2)}$ reads

$$\left(\mathcal{D}^{(0)2}A^{\dagger(2)}\right)_{\dot a} = \frac{32k^2\rho^4C^2q^\dagger_\alpha x^\alpha_{\dot a}\bar\zeta^2\eta^2}{(x^2+\rho^2)^{9/2}} - \frac{64k^2\rho^2C^2q^\dagger_\alpha x^\alpha_{\dot a}\bar\zeta^2\eta^2}{(x^2+\rho^2)^{9/2}}(2x^2-\rho^2) -$$

$$-\frac{8h\rho^2C^2q^\dagger_\alpha x^\alpha_{\dot a}\bar\zeta^2\eta^2}{(x^2+\rho^2)^{9/2}}\left((2h-5k)x^2+(6h+3k)\rho^2\right). \tag{B.29}$$

The convolution with (A.9) yields

$$A^{\dagger(2)}_{\dot a} = \frac{C^2q^\dagger_\alpha x^\alpha_{\dot a}\bar\zeta^2\eta^2}{(x^2+\rho^2)^{5/2}}\left((-6k^2+2hk-2h^2)x^2+(k^2-3h^2)\rho^2\right). \tag{B.30}$$

In (6.25) we report the explicit solution with $k=1=h$.

The equation for $\psi^{(2)}$ drops out as already explained in the main text.

C. Table of integrals

In solving our systems of coupled differential equations the following integrals turn out to be very useful

$$\mathcal{I}^p_{(0)} \equiv \int d^4y\frac{1}{(y^2+\rho^2)^p(x-y)^2} = \frac{i\pi^2}{p-1}\int_0^1 dz\frac{1}{(\rho^2+zx^2)^{p-1}}$$

$$\mathcal{I}^p_{(1)\alpha\dot\alpha} \equiv \int d^4y\frac{y_{\alpha\dot\alpha}}{(y^2+\rho^2)^p(x-y)^2} = \frac{i\pi^2}{p-i}\int_0^1 dz\frac{zx_{\alpha\dot\alpha}}{(\rho^2+zx^2)^{p-1}}$$

$$\mathcal{I}^p_{(2)\alpha\dot\alpha\beta\dot\beta} \equiv \int d^4y\frac{y_{\alpha\dot\alpha}y_{\beta\dot\beta}}{(y^2+\rho^2)^p(x-y)^2}$$

$$= \frac{i\pi^2}{p-1}\int_0^1 dz\left[\frac{(1-z)\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}}{(p-2)(\rho^2+zx^2)^{p-2}}+\frac{z^2x_{\alpha\dot\alpha}x_{\beta\dot\beta}}{(\rho^2+zx^2)^{p-1}}\right]$$

$$\mathcal{I}^p_{(3)\alpha\dot\alpha\beta\dot\beta\gamma\dot\gamma} \equiv \int d^4y\frac{y_{\alpha\dot\alpha}y_{\beta\dot\beta}y_{\gamma\dot\gamma}}{(y^2+\rho^2)^p(x-y)^2}$$

$$= \frac{i\pi^2}{p-1}\int_0^1 dz\left[\frac{z(1-z)(\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}x_{\gamma\dot\gamma}+\text{perm.})}{(p-2)(\rho^2+zx^2)^{p-2}}+\frac{z^3x_{\alpha\dot\alpha}x_{\beta\dot\beta}x_{\gamma\dot\gamma}}{(\rho^2+zx^2)^{p-1}}\right]. \tag{C.1}$$

These are the prototypes for all other integrals used in this paper, which can be obtained using the following recursion formulae

$$\begin{aligned}
\mathcal{I}^{p}_{(2k)} &\equiv \int d^4y \frac{(y^2)^k}{(y^2+\rho^2)^p(x-y)^2} = \mathcal{I}^{p-1}_{(2k-2)} - \rho^2 \mathcal{I}^{p}_{(2k-2)} \\
\mathcal{I}^{p}_{(2k+1)\alpha\dot\alpha} &\equiv \int d^4y \frac{y_{\alpha\dot\alpha}(y^2)^k}{(y^2+\rho^2)^p(x-y)^2} = \mathcal{I}^{p-1}_{(2k-1)\alpha\dot\alpha} - \rho^2 \mathcal{I}^{p}_{(2k-1)\alpha\dot\alpha} \\
\mathcal{I}^{p}_{(2k+2)\alpha\dot\alpha\beta\dot\beta} &\equiv \int d^4y \frac{y_{\alpha\dot\alpha}y_{\beta\dot\beta}(y^2)^k}{(y^2+\rho^2)^p(x-y)^2} = \mathcal{I}^{p-1}_{(2k)\alpha\dot\alpha\beta\dot\beta} - \rho^2 \mathcal{I}^{p}_{(2k)\alpha\dot\alpha\beta\dot\beta}\,.
\end{aligned} \tag{C.2}$$

References

[1] J.H. Schwarz and P.Van Nieuwenhuizen, *Speculations concerning a fermionic substructure of space-time*, *Lett. Nuovo Cim.* **34** (1982) 21;
P. Bouwknegt, J.G. McCarthy and P. van Nieuwenhuizen, *Fusing the coordinates of quantum superspace*, *Phys. Lett.* **B 394** (1997) 82 [hep-th/9611067];
S. Ferrara and M.A. Lledo, *Some aspects of deformations of supersymmetric field theories*, *JHEP* **05** (2000) 008 [hep-th/0002084];
D. Klemm, S. Penati and L. Tamassia, *Non(anti)commutative superspace*, *Class. and Quant. Grav.* **20** (2003) 2905 [hep-th/0104190].

[2] N. Seiberg, *Noncommutative superspace, $N = 1/2$ supersymmetry, field theory and string theory*, *JHEP* **06** (2003) 010 [hep-th/0305248].

[3] R. Britto, B. Feng and S.-J. Rey, *Deformed superspace, $N = 1/2$ supersymmetry and (non)renormalization theorems*, *JHEP* **07** (2003) 067 [hep-th/0306215];
S. Terashima and J.T. Yee, *Comments on noncommutative superspace*, *JHEP* **12** (2003) 053 [hep-th/0306237];
M.T. Grisaru, S. Penati and A. Romagnoni, *Two-loop renormalization for nonanticommutative $N = 1/2$ supersymmetric WZ model*, *JHEP* **08** (2003) 003 [hep-th/0307099];
R. Britto and B. Feng, *$N = 1/2$ Wess-Zumino model is renormalizable*, *Phys. Rev. Lett.* **91** (2003) 201601 [hep-th/0307165];
A. Romagnoni, *Renormalizability of $N = 1/2$ Wess-Zumino model in superspace*, *JHEP* **10** (2003) 016 [hep-th/0307209];
O. Lunin and S.J. Rey, *Renormalizability of non(anti)commutative gauge theories with $N = 1/2$ supersymmetry*, *JHEP* **09** (2003) 045 [hep-th/0307275];
D. Berenstein and S.J. Rey, *Wilsonian proof for renormalizability of $N = 1/2$ supersymmetric field theories*, *Phys. Rev.* **D 68** (2003) 121701 [hep-th/0308049];
M. Alishahiha, A. Ghodsi and N. Sadooghi, *One-loop perturbative corrections to non(anti)commutativity parameter of $N = 1/2$ supersymmetric $\mathrm{U}(N)$ gauge theory*, *Nucl. Phys.* **B 691** (2004) 111 [hep-th/0309037];
I. Jack, D.R.T. Jones and L.A. Worthy, *One-loop renormalisation of $N = 1/2$ supersymmetric gauge theory*, *Phys. Lett.* **B 611** (2005) 199 [hep-th/0412009].

[4] J. de Boer, P.A. Grassi and P. van Nieuwenhuizen, *Non-commutative superspace from string theory*, *Phys. Lett.* **B 574** (2003) 98 [hep-th/0302078];
H. Ooguri and C. Vafa, *The c-deformation of gluino and non-planar diagrams*, *Adv. Theor. Math. Phys.* **7** (2003) 53 [hep-th/0302109];

H. Ooguri and C. Vafa, *Gravity induced c-deformation*, *Adv. Theor. Math. Phys.* **7** (2004) 405 [hep-th/0303063].

[5] N. Berkovits and N. Seiberg, *Superstrings in graviphoton background and $N = 1/2 + 3/2$ supersymmetry*, *JHEP* **07** (2003) 010 [hep-th/0306226].

[6] M. Billo, M. Frau, I. Pesando and A. Lerda, *$N = 1/2$ gauge theory and its instanton moduli space from open strings in RR background*, *JHEP* **05** (2004) 023 [hep-th/0402160];
M. Billo, M. Frau, F. Lonegro and A. Lerda, *$N = 1/2$ quiver gauge theories from open strings with RR fluxes*, *JHEP* **05** (2005) 047 [hep-th/0502084].

[7] A. Imaanpur, *On instantons and zero modes of $N = 1/2$ SYM theory*, *JHEP* **09** (2003) 077 [hep-th/0308171].

[8] A. Imaanpur, *Comments on gluino condensates in $N = 1/2$ SYM theory*, *JHEP* **12** (2003) 009 [hep-th/0311137].

[9] P.A. Grassi, R. Ricci and D. Robles-Llana, *Instanton calculations for $N = 1/2$ super Yang-Mills theory*, *JHEP* **07** (2004) 065 [hep-th/0311155].

[10] R. Britto, B. Feng, O. Lunin and S.J. Rey, U(N) *instantons on $N = 1/2$ superspace: exact solution and geometry of moduli space*, *Phys. Rev.* **D 69** (2004) 126004 [hep-th/0311275].

[11] T. Araki, K. Ito and A. Ohtsuka, *Supersymmetric gauge theories on noncommutative superspace*, *Phys. Lett.* **B 573** (2003) 209 [hep-th/0307076].

[12] N. Dorey, T.J. Hollowood, V.V. Khoze and M.P. Mattis, *The calculus of many instantons*, *Phys. Rept.* **371** (2002) 231 [hep-th/0206063].

[13] L.G. Aldrovandi, D.H. Correa, F.A. Schaposnik and G.A. Silva, *BPS analysis of gauge field — Higgs models in non- anticommutative superspace*, *Phys. Rev.* **D 71** (2005) 025015 [hep-th/0410256].

[14] I. Affleck, *On constrained instantons*, *Nucl. Phys.* **B 191** (1981) 429;
M. Nielsen and N.K. Nielsen, *Explicit construction of constrained instantons*, hep-th/9912006.

[15] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, *Nonperturbative aspects in supersymmetric gauge theories*, *Phys. Rept.* **162** (1988) 169.

[16] M.A. Shifman and A.I. Vainshtein, *Instantons versus supersymmetry: fifteen years later*, hep-th/9902018.

[17] T. Hatanaka, S.V. Ketov, Y. Kobayashi and S. Sasaki, *Non-anti-commutative deformation of effective potentials in supersymmetric gauge theories*, *Nucl. Phys.* **B 716** (2005) 88 [hep-th/0502026].

[18] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in supersymmetric QCD*, *Nucl. Phys.* **B 241** (1984) 493.

[19] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in four-dimensions and its phenomenological implications*, *Nucl. Phys.* **B 256** (1985) 557.

[20] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Supersymmetric instanton calculus: gauge theories with matter*, *Nucl. Phys.* **B 260** (1985) 157.

JHEP10(2005)021